# Consumer search with blind buying ${ }^{\pi}$ 

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## A R T I C L E I N F O

## Article history:

Received 25 February 2020
Available online 28 January 2021

## JEL classification:

D8
L1

## Keywords:

Consumer search
Blind buying
Observable price
Search cost
Prominence


#### Abstract

This article studies a sequential search model in which consumers can purchase a product without incurring a search cost to inspect the match value, which we call "blind buying". We show that the optimal search policy is no longer as per Weitzman (1979). When the match value has a symmetric distribution, both consumers and firms are indifferent to the search order, conditional on that blind buying does not take place in the first stage. Blind buying always increases total welfare, and increases market prices and industrial profits if and only if the first-sample search cost is below a threshold value. An increase in the search cost reduces equilibrium prices. Such a result is consistent with existing price-directed search models, but the underlying mechanisms are different. We also show that being prominent can adversely affect a firm if the match value is asymmetrically distributed, which contrasts the literature.


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## 1. Introduction

While shopping, consumers do not typically have all relevant information and must invest effort in order to purchase a suitable product at a reasonable price. Therefore, economists have developed sequential consumer search models to study such situations. In traditional search models, such as Wolinsky's (1986) and Anderson and Renault's (1999), the typical assumption is that consumers pay a fixed search cost to visit a store and then become fully informed about the price and match value at that store. Particularly, consumers cannot buy a product unless they pay the search cost. Such an assumption is reasonable for offline shopping, in which the fixed search cost can be expressed as the transportation cost of visiting a store. Once consumers arrive the store, the price and match value of a product are relatively easy to observe, because they can touch, feel, and try the product. It is therefore reasonable to assume that, after arriving at the store, the cost of learning the price and match value is zero. In such scenarios, it is natural to assume that consumers cannot buy a product if they have not visited a store.

However, online shopping brings new features that traditional search models cannot capture. On one hand, the highly developed online search technologies have significantly lowered the cost of visiting stores and collecting information about prices. For online shopping, the costs of visiting a store and observing prices are almost negligible. On the other hand,

[^0]consumers cannot physically inspect and try products and thus have to exert a costly effort by, for example, reviewing product descriptions, images, and other consumers' feedback, to evaluate the match value of a product. Therefore, search cost should be interpreted as the cost required to accurately learn a product's match value, rather than the transportation cost of visiting a store. The literature has already taken notice of this fact. For instance, Choi et al. (2018) developed a search model in which consumers can observe prices at no cost, but incur a search cost to fully observe the match value. Although they interpret the search cost as the cost of learning the match value, they maintain the assumption that consumers cannot buy a product unless they pay the search cost. Under this interpretation of search costs, it is theoretically more reasonable to assume consumers have the option to buy the product without making an effort to learn the match value.

This article thus develops a sequential search model to capture these new features. The basic model is a duopoly version of Choi et al.'s (2018) model, except that we allow consumers to buy a product without inspecting the match value. For tractability, we also assume the distribution of the match value to be symmetric. Before searching, all consumers observe the prices and prior values of both firms, but have to pay a search cost to learn the match value. Consumers have to decide which product to search firstly and whether to continue searching after inspecting the first product. If consumers decide to stop searching, they can either buy the inspected product or blindly buy an uninspected product, the latter option being new to the literature. We analyze how this blind buying option affects consumers' searching behavior, firms' pricing strategies, and their incentives to become prominent.

Before conducting the equilibrium analysis, we present the following online-and-offline example that fits our setup of the blind buying. With the deep integration of the online and offline channels, an increasing number of firms adopt the multi-channel (brick-and-click) approach. Examples include well-known cases such as Walmart, Macy's, Staples. Consumers can either shop online, without visiting a firm's physical store, or make a purchase decision after visiting the firm's physical store. Visiting the physical store is costly, but allows consumers to learn detailed information about a product's match value. Buying directly online can help consumers save the costs of visiting the physical store, but they can only obtain inaccurate information about the match value. In this example, the blind buying is shopping directly online, while the informed buy means buying the product after visiting the physical store.

We assume that the search cost is $c_{1}$ for the first sample and $c_{2}$ for the second one. Our setup incorporates the standard case where $c_{1}=c_{2}=c$, as well as the assumption that the first sample is free, i.e., $c_{1}=0$. In reality, a consumer's search costs for different samples need not be the same. For example, a consumer's patience may be wearing thin as she samples more products, so the search cost per sample may increase during the search. Another example is "learning by doing": a consumer may gain more experience in searching, so the search cost per sample may decrease during the search.

Without the blind buying option, consumers' optimal searching strategy is fully described by Weitzman's (1979) model, which consists of a search order and a stopping rule. Since we assume that the distributions of the two products' match values are identical, a consumer first searches the product with a higher net prior value (equal to the prior value minus the price). The consumer buys the product if the match value of the first search exceeds a threshold value, or continues to search if the match value is below the threshold value.

Allowing for blind buying significantly changes consumers' searching behavior. Weitzman's (1979) optimal policy is no longer applicable. In the first stage, due to the first-sample search cost, a consumer will blindly buy a product without entering the search process when the difference of the products' prior values is large. Sampling takes place only when the prior value difference is small. Conditional on searching in the first stage, the optimal stopping rule in the second stage consists of two threshold values: upper and lower reservation values. If the observed match value exceeds the upper reservation value, the consumer stops searching and buys the inspected product. If the match value is below the lower reservation value, the consumer stops the search and blindly buys the uninspected product. Therefore, while the upper reservation value ends the search process by approving a product that is good enough (as in existing search models), the lower reservation value, introduced by blind buying, can also stop consumers from searching by eliminating an unsatisfactory product. The consumer continues to search only if the observed match value is between the two reservation values.

With the option of blind buying, the optimal search order of a consumer no longer corresponds to Weitzman's (1979). We show that the consumer is indifferent among search orders, conditional on that blind buying does not take place in the first stage. The intuition is as follows. Since searching is costly, the consumer wants to end the search process as early as possible. Without a blind buying option, the search ends only when a product is discovered to be good enough. Therefore, the most efficient way is to first search the product with the highest net prior value, because this product is most likely to exceed the upper reservation utility and get approved. In contrast, with the blind buying option, the search can be terminated either by a product approval, when the inspected match value is above the upper reservation value, or by a product elimination, when the inspected match value is below the lower reservation value. The consumer may want to first sample the product with a lower net prior value because it is more likely to be eliminated. Since the match value is distributed symmetrically, the effect of product elimination offsets the effect of product approval, leaving consumers indifferent to the search order.

We also show that firms' profits are independent of consumers' search order. As a result, ranking first in the search order is irrelevant for a firm. Being sampled first poses both a benefit and a risk for firms. The benefit is that it will be selected without further search when the match value exceeds the upper reservation value; and the risk is that it will be eliminated from consumers' choice set without further search when the match value is below the lower reservation value. Because the match value is symmetrically distributed, the benefit and risk offset each other, so that firms are indifferent to the search order.

The effect of blind buying on equilibrium prices can be either positive or negative. On the one hand, conditional on searching in the first stage, firms' profits are independent of consumers' search order. Thus, a firm has less incentive to lower its price in order to attract first visits. Consequently, blind buying has positive effect on equilibrium prices. On the other hand, the option of blind buying allows consumers to make uninformed purchasing decisions, i.e., purchase without inspecting any products. That consumers possess less information about their tastes reduces products' heterogeneity and increases price competition. Thus, blind buying has negative effect on equilibrium prices. The former effect dominates and blind buying increases equilibrium prices if and only if the first-sample search cost is small.

We show that, with blind buying, an increase in the first-sample search cost reduces equilibrium prices, while it has no effect on market prices when blind buying is not allowed. The effect of the second-sample search cost on market prices is the same as in existing price-directed search models, but the underlying mechanisms are different. In existing pricedirected search models, a price cut can attract more consumers to visit a firm first, and these first visits are more likely to lead to final sales as search costs increase. As a result, firms' incentives to lower their prices increase with the search costs. This mechanism does not exist in our blind buying model. First, a price reduction does not attract more first samplings, as consumers are indifferent to the search order. Second, even if a firm successfully attracts more consumers for first samplings, its profit does not increase, as profits are independent of consumers' search order. Instead, the blind buying option creates a new effect on price competition, the so-called blind-buying effect. In each stage of the search process, a blind buying takes place whenever a product is unsatisfactory compared to its competitor and is therefore eliminated. Thus, firms have incentives to lower their prices in order to attract more blind buying, making the market more competitive.

Our welfare analysis shows that, blind buying always increases total welfare. Blind buying offers consumers an additional way to end the search process, which saves total search costs and thus increases the overall social welfare. If the first-sample search cost is high, blind buying also reduces the equilibrium prices and thus increases consumer surplus. However, if the first-sample search cost is low, blind buying lowers consumer search costs on one hand and leads to higher equilibrium prices on the other, leaving an ambiguous effect on consumer surplus.

We then investigate the case in which the match value is distributed asymmetrically. To ensure tractability, we assume the distribution of the match value is binary, that is, either high or low. We first examine the case in which the search order is exogenous and independent of the product prices. Such an assumption is reasonable if we assume that the cost of visiting one firm is sufficiently lower than that of visiting the other, which is the case when one firm opens its offline store in a convenient location while the other opens its offline store in an unfavorable location in our aforementioned online-and-offline example. Surprisingly, we show that being prominent can be harmful to a firm, which is in sharp contrast to the literature. Intuitively, if the match value is likely low, the product in the non-prominent position will most likely be blindly purchased by consumers, resulting in higher profits for the producing firm. In such cases, a firm may want to pay to avoid being prominent. Empirically, most of the evidence supports it is beneficial to appear earlier in the search list (Ellison and Ellison, 2009; Baye et al., 2009; Brynjolfsson et al., 2010, and the citations therein), but there is also evidence that it can be beneficial to appear later (Novarese and Wilson, 2013; De los Santos and Koulayev, 2013; Feenberg et al., 2017). Our model provides a possible explanation for the mixed evidence of position effects. We then investigate the case where consumers can endogenously select the search order. We show that consumers may search the product with a lower net prior value first, so an increase in price can attract more first samplings. In this case, however, firms will be harmed if they are sampled first. Consequently, firms have incentives to lower their prices so as not to be sampled first.

Our work belongs to the consumer search literature on non-random sequential searches. The first-generation models examine a setup where the search order is exogenously fixed. For instance, Arbatskaya (2007) studies a market of homogeneous goods in which consumers search for better prices. Subsequent articles take horizontal product differentiation into account, so that consumers search for both price and product fitness. Armstrong et al. (2009) and Armstrong and Zhou (2011) examine the possibility that one firm is prominent, meaning all consumers search this firm first and then randomly search the remaining firms. Zhou (2011) studies the case in which there is a complete exogenous search order, finding that higher ranking leads to higher profits in general, and firms prefer to be visited first. The only exception is Fishman and Lubensky (2018a), who introduce costly returns into a search model and show that appearing later in the search could be preferable. The reason is that costly returns benefit a later searched seller by preventing returns conditional on the search. Our model offers another rationale why prominence is not necessarily beneficial.

The second-generation models endogenize the search order. The work-horse model is Weitzman's (1979), in which an agent named Pandora is presented with boxes containing prizes and can open the boxes in any order, discover the prizes within, and optimally stop. The optimal policy, so-called Pandora's rule, specifies a stopping rule and a remarkably simple search order. Following Weitzman (1979), several articles have recently investigated price-directed consumer searches. Motivated by the fact that price information is easy to obtain online, these papers assume all prices are observable before the search and consumers endogenously select their search order according to prices and pre-search information (see, e.g., Armstrong and Zhou, 2011; Armstrong, 2017; Haan et al., 2018; Choi et al., 2018; Ding and Zhang, 2018). In contrast to traditional random search models, these models find that an increase in search costs leads to a decrease in equilibrium prices. Intuitively, consumers always visit the firm with the lowest price and tend to buy from the first one with increasing search costs. As a result, firms are in fierce competition to be visited first and thus lower their prices. The main difference between our article and the literature is that we allow consumers to buy blindly. We show that the equilibrium in our model differs substantially from that in existing price-directed search models.

Our article also contributes to the literature in search models in which searchers can endogenously decide whether to be informed before decide on an object. Doval's (2018) is the first study that extends Weitzman (1979)'s model by allowing searchers to take a box without inspecting it and provides sufficient conditions to characterize the optimal policy. Other papers investigate the situation that an agent has to choose an object with multi attributes and can discover the quality of each attribute at a cost. The problem is how many attributes to investigate and whether to take the object based on partial information (Branco et al., 2012; Klabjan et al., 2014; Olszewski and Wolinsky, 2013; Ke et al., 2016; Ke and VillasBoas, 2018). However, these articles focus mainly on searchers' behaviors and do not consider firms' strategic behaviors. ${ }^{1}$ By contrast, we are also interested in firms' strategic pricing decisions and their incentives to become prominent. There are two studies that incorporate firms' pricing decisions. First, Fishman and Lubensky (2018b) construct a model in which consumers first pay a transportation cost to discover a seller's price and then have the option to pay a search cost to learn the product's match value before deciding to buy or continue searching. Second, Chen et al. (2018) allow consumers to choose the depth of evaluation before buying a product. A common and interesting result of these two articles is that equilibrium prices and consumer surplus are both non-monotonic in the search cost. The topic of our article is quite similar to theirs, with the difference that while these two articles consider a random search, we examine the case of a directed search. As a result, we are also able to discuss the search order, which is not possible within their framework.

The rest of the paper is organized as follows. We introduce the formal model in Section 2, analyzing consumers' optimal search strategy and characterizing market equilibrium. In Section 3, we first study how the presence of a blind buying option affects the relationship between market prices and search costs. Then, we investigate the effect of the blind buying assumption on welfare levels. In Section 4, we study the effect of prominence when a blind buying option is available, and show that prominence could be harmful to a firm. Section 5 concludes the paper. All proofs are given in the appendix.

## 2. The model

Consider a duopoly market, in which two firms sell horizontally differentiated products in price competition. Production costs are normalized to zero. The demand side of the market consists of a unit mass of consumers, each of whom is riskneutral and wants to buy one unit of the products. ${ }^{2}$ Firms announce prices simultaneously. Consumers costlessly observe products' prices and search optimally for the best product. The search is assumed to be sequential and with perfect recall.

Following Choi et al. (2018) and Haan et al. (2018), the random utility (net of any search cost) of consumer l, who purchases the product from firm $i=1,2$, is given by:

$$
\tilde{u}_{i l}=\tilde{\eta}_{i l}+\tilde{\varepsilon}_{i l}-p_{i}
$$

where $p_{i}$ is product $i$ 's price; random variable $\tilde{\eta}_{i l}$ represents the observable match value of product $i$ to consumer $l$, which can be interpreted as the pre-search information of the product value that can be costlessly observed by consumers; and random variable $\tilde{\varepsilon}_{i l}$ represents the opaque match value between product $i$ and consumer $l$, which is unobservable but can be discovered at some search cost. To distinguish the two types of match values, we simply call $\tilde{\eta}_{i l}$ the "prior value" and $\tilde{\varepsilon}_{i l}$ the "match value" throughout this paper.

Let $\eta_{i l}$ and $\varepsilon_{i l}$ denote the realized values of random variables $\tilde{\eta}_{i l}$ and $\tilde{\varepsilon}_{i l}$, respectively. We assume that $\tilde{\eta}_{i l}$ and $\tilde{\varepsilon}_{i l}$ are independently and identically distributed across consumers (l's) and products ( $i$ 's), according to distribution functions $G$ and $F$, respectively. Additionally, $\tilde{\eta}_{i l}$ and $\tilde{\varepsilon}_{i l}$ are independent of each other.

As previously mentioned, the key feature of our model is that we allow consumers to blindly buy a product. That is, at any stage of the search process, consumers have the option of purchasing a product without inspecting its match value $\tilde{\varepsilon}_{i l}$, in which case no search cost is incurred. If consumer $l$ blindly buys product $i$, that is, the consumer learns ( $\eta_{i l}, p_{i}$ ) but not $\varepsilon_{i l}$, the expected utility (net of search costs) for consumer $l$ is

$$
E \tilde{u}_{i l}=\eta_{i l}+E\left[\tilde{\varepsilon}_{i l}\right]-p_{i}
$$

Search is assumed to be costly. We assume that the search cost is $c_{1}$ for the first sample and $c_{2}$ for the second one. Our setup is general and includes the standard case where $c_{1}=c_{2}=c$, as well as the case where the first sample is free, i.e., $c_{1}=0$. As we will show later, with the option of "blind buying", the first-sample search cost $c_{1}$ and the second-sample search cost $c_{2}$ have different effects on the players' strategies as well as the market outcomes.

Allowing blind buying complicates the consumer search behavior. On one hand, consumers have to decide at every stage of the search process whether they want to buy a product blindly. On the other hand, the optimal search order becomes more subtle when consumers expect a blind buying to take place in the future. The analysis of our model is somewhat more difficult, since the optimal search strategy of each consumer, as we show later, is no longer completely characterized by Weitzman's (1979) solution.

We assume that the distribution function of $\tilde{\eta}_{i l}, G$, has a continuously differentiable density function, $g=G^{\prime}$, with infinite support. We also assume that the density function $g$ is log-concave.

[^1]In the main model, we assume that the match value $\tilde{\varepsilon}_{i l}$ is distributed symmetrically and continuously with infinite support ${ }^{3}$ and density function $f=F^{\prime}$ is continuously differentiable and log-concave. Log-concavity of $f$ and $g$ is commonly assumed in the consumer search literature. This assumption not only guarantees the existence and uniqueness of a market equilibrium, but also generates clear-up comparative statics results.

In Section 4, we relax the assumption that $\tilde{\varepsilon}_{i l}$ is symmetrically distributed. In general, the optimal search policy is difficult to characterize when $\tilde{\varepsilon}_{i l}$ is not symmetric. For traceability, we focus on the case where $\tilde{\varepsilon}_{i l}$ has a binary distribution (which need not be symmetric). We solve the market equilibrium and investigate whether prominence is always beneficial for firms.

The game proceeds as follows. In stage 0 , firms simultaneously announce prices $\left(p_{1}, p_{2}\right)$. In stage 1 , each consumer costlessly observes both the prices and the prior values of the products ( $p_{1}, p_{2}, \eta_{1}, \eta_{2}$ ) ${ }^{4}$ and decides whether to blindly buy a product. The game ends if a blind buying takes place, or moves to stage 2 if the consumer chooses to discover a product's match value. In stage 2 , after learning product $i$ 's match value $\tilde{\varepsilon}_{i}, i \in\{1,2\}$, the consumer decides among three options: (i) buy product $i$ with no uncertainty; (ii) blindly buy product $j$ without inspecting $\tilde{\varepsilon}_{j}$, where $j \in\{1,2\} \backslash\{i\}$; and (iii) discover the match value $\varepsilon_{j}$ and purchase the product with a higher realized utility. We first study the optimal search strategy of the consumer and then solve the pricing game between the two firms.

### 2.1. Optimal consumer search behavior

We first consider consumers' search behaviors. Suppose a representative consumer has already observed the product prices and prior values $\left(p_{1}, p_{2}, \eta_{1}, \eta_{2}\right)$. We define $v_{i} \equiv \eta_{i}-p_{i}$ as product $i$ 's net prior value and $\Delta_{i} \equiv v_{i}-v_{j}$ as the difference between the net prior values of products $i$ and $j$, for $i=1,2$. Note that $\Delta_{i}=-\Delta_{j}$ for $i \neq j$.

The consumer will obtain a payoff $u_{0}=\max \left\{v_{1}, v_{2}\right\}+E \tilde{\varepsilon}$ if buying blindly in stage 1 . Alternatively, the consumer can choose to discover a product's match value and proceed to stage 2 . Let $u_{i}\left(v_{1}, v_{2}\right)$ be the consumer's expected continuation payoff (net of search costs) in stage 2 , given that she previously inspected product $i$ in stage 1 , for $i=1$, 2 . Thus, the maximization problem at stage 1 becomes:

$$
\begin{equation*}
\max \left\{u_{0}, u_{1}\left(v_{1}, v_{2}\right)-c_{1}, u_{2}\left(v_{1}, v_{2}\right)-c_{1}\right\} \tag{1}
\end{equation*}
$$

where $c_{1}$ is the first-sample search cost.
Now we use backward induction to solve $u_{i}\left(v_{1}, v_{2}\right)$. Assume that the consumer observes $\left(p_{1}, p_{2}, \eta_{1}, \eta_{2}, \varepsilon_{i}\right)$ at the beginning of stage 2 . The payoff is $v_{i}+\varepsilon_{i}$ if she purchases product $i$ immediately, $v_{j}+E \tilde{\varepsilon}$ if buys product $j$ blindly and $-c_{2}+E_{\tilde{\varepsilon}_{j}} \max \left(v_{i}+\varepsilon_{i}, v_{j}+\tilde{\varepsilon}_{j}\right)$ if continues to inspect $\tilde{\varepsilon}_{j}$ and purchases the best product, where $c_{2}$ is the second-sample search cost. Therefore, based on the information ( $p_{1}, p_{2}, \eta_{1}, \eta_{2}, \varepsilon_{i}$ ), the consumer's maximized payoff in stage 2 is

$$
\begin{align*}
u_{i}\left(v_{1}, v_{2}, \varepsilon_{i}\right) & =\max \left\{v_{i}+\varepsilon_{i}, v_{j}+E \tilde{\varepsilon},-c_{2}+E_{\tilde{\varepsilon}_{j}} \max \left(v_{i}+\varepsilon_{i}, v_{j}+\tilde{\varepsilon}_{j}\right)\right\} \\
& =v_{j}+\max \left\{\Delta_{i}+\varepsilon_{i}, E \tilde{\varepsilon},-c_{2}+E_{\tilde{\varepsilon}_{j}} \max \left(\Delta_{i}+\varepsilon_{i 1}, \tilde{\varepsilon}_{j}\right)\right\} \tag{2}
\end{align*}
$$

The expected continuation payoffs are thus given as follows:

$$
\begin{equation*}
u_{i}\left(v_{1}, v_{2}\right)=E_{\tilde{\varepsilon}_{i}} u_{i}\left(v_{1}, v_{2}, \tilde{\varepsilon}_{i}\right), \text { for } i=1,2 \tag{3}
\end{equation*}
$$

Since match value $\tilde{\varepsilon}_{i}$ is symmetrically and continuously distributed, $i=1,2$, we have:

$$
\begin{equation*}
E \tilde{\varepsilon}=0, F(-t)=1-F(t) \text { and } f(t)=f(-t), \forall t \tag{4}
\end{equation*}
$$

We define

$$
H(x) \equiv-c_{2}+E_{\tilde{\varepsilon}} \max (x, \tilde{\varepsilon})=-c_{2}+x+\int_{x}^{\infty}(t-x) d F(t)
$$

which is the expected continuation payoff of a consumer that chooses to inspect the next product instead of choosing current option $x$. Using condition (4) and the definition of $H(\cdot)$, we can rewrite expression (2) as

$$
\begin{equation*}
u_{i}\left(v_{1}, v_{2}, \varepsilon_{i}\right)=v_{j}+\max \left\{\Delta_{i}+\varepsilon_{i}, 0, H\left(\Delta_{i}+\varepsilon_{i}\right)\right\} \tag{5}
\end{equation*}
$$

Note that $H^{\prime}(x)=F(x) \in(0,1)$, for any $x \in(-\infty, \infty)$. This implies (i) the consumer's incremental utility between inspecting the next product and purchasing the sampled product, $H(x)-x$, decreases with current option $x$; and (ii) the incremental utility between inspecting and blindly buying the next product, $H(x)$, increases with $x$.

[^2]Let $\hat{\varepsilon}_{h}$ and $\hat{\varepsilon}_{l}$ be the unique solution to the following two equations, respectively:

$$
\begin{align*}
& H\left(\hat{\varepsilon}_{h}\right)=\hat{\varepsilon}_{h}, \text { i.e., } c_{2}=\int_{\hat{\varepsilon}_{h}}^{\infty}\left(t-\hat{\varepsilon}_{h}\right) d F(t)  \tag{6}\\
& H\left(\hat{\varepsilon}_{l}\right)=0, \text { i.e., } c_{2}=\hat{\varepsilon}_{l}+\int_{\hat{\varepsilon}_{l}}^{\infty}\left(t-\hat{\varepsilon}_{l}\right) d F(t) \tag{7}
\end{align*}
$$

Therefore, $\hat{\varepsilon}_{h}$ is the traditional reservation utility that makes the consumer indifferent between continuing sampling and purchasing immediately. The option of blind buying introduces a new reservation utility, $\hat{\varepsilon}_{l}$, at which the consumer is indifferent between continuing sampling and blindly buying the uninspected product. The symmetry of $F$ implies that $\hat{\varepsilon}_{l}=-\hat{\varepsilon}_{h} .{ }^{5}$

To ensure that the search ever takes place in both stages, we assume that the search costs $c_{1}$ and $c_{2}$ are not too high. ${ }^{6}$
Assumption 1. The search costs satisfy $0<c_{1}, c_{2}<\int_{0}^{\infty} t d F(t)$.
Under Assumption 1, the two cutoff values $\hat{\varepsilon}_{h}$ and $\hat{\varepsilon}_{l}$ satisfy that $\hat{\varepsilon}_{l}<0<\hat{\varepsilon}_{h}$. We call $\hat{\varepsilon}_{h}$ and $\hat{\varepsilon}_{l}$ the upper and lower reservation utilities, respectively. The optimal search behavior in stage 2 is given by the following proposition.

Proposition 1. Assume that the match value is symmetrically and continuously distributed and Assumption 1 holds. If the consumer previously sampled product $i$ and observed $\varepsilon_{i}$ in stage 1, the optimal search strategy in stage 2 is to
(a) buy product if $\Delta_{i}+\varepsilon_{i} \geq \hat{\varepsilon}_{h}$;
(b) blindly buy product $j$ if $\bar{\Delta}_{i}+\varepsilon_{i}<-\hat{\varepsilon}_{h}$;
(c) continue to discover $\tilde{\varepsilon}_{j}$ if $-\hat{\varepsilon}_{h} \leq \Delta_{i}+\varepsilon_{i}<\hat{\varepsilon}_{h}$.

The consumer will stop searching and make a purchase when the current option is either above $\hat{\varepsilon}_{h}$ or below $\hat{\varepsilon}_{l}$. In the former case, the consumer will purchase the current sampled product, considering it good enough. In the latter case, the consumer simply eliminates the sampled product and opts for the other product without sampling. Therefore, while the upper reservation utility $\hat{\varepsilon}_{h}$ ends the search process by approving a product (as in existing search models), the lower reservation utility $\hat{\varepsilon}_{l}$ introduced by blind buying can also stop consumers from searching by product elimination. ${ }^{7}$ When the difference of the products is small (i.e., $-\hat{\varepsilon}_{h} \leq \Delta_{i}+\varepsilon_{i}<\hat{\varepsilon}_{h}$ ), the sampled product cannot easily be approved or eliminated, so that the consumer must continue sampling in order for a purchase decision.

Given the optimal search behavior in stage 2 , we can solve expected continuation payoffs $u_{i}\left(v_{1}, v_{2}\right)$ by substituting (5) into (3):

$$
u_{i}\left(v_{1}, v_{2}\right)=\int_{-\infty}^{-\hat{\varepsilon}_{h}-\Delta_{i}} v_{j} d F\left(\varepsilon_{i}\right)+\int_{-\hat{\varepsilon}_{h}-\Delta_{i}}^{\hat{\varepsilon}_{h}-\Delta_{i}}\left[H\left(\Delta_{i}+\varepsilon_{i}\right)+v_{j}\right] d F\left(\varepsilon_{i}\right)+\int_{\hat{\varepsilon}_{h}-\Delta_{i}}^{\infty}\left(v_{i}+\varepsilon_{i}\right) d F\left(\varepsilon_{i}\right)
$$

where the first term of the right-hand side is the utility from blindly buying product $j$, the second term is the utility from continuing sampling product $j$, and the third term is the utility from buying product $i$ immediately.

$$
\begin{aligned}
& 5 \text { The symmetry of } F \text { implies that for any } x, F(x)=1-F(-x) \text { and } \int_{-x}^{x} t d F(t)=0 \text {. Thus } \\
& \qquad \begin{aligned}
\int_{x}^{\infty}(t-x) d F(t) & =\int_{x}^{\infty} t d F(t)-x[1-F(x)] \\
& =\int_{-x}^{\infty} t d F(t)-x+x[1-F(-x)] \\
& =-x+\int_{-x}^{\infty}(t+x) d F(t)
\end{aligned}
\end{aligned}
$$

[^3]Lemma 1. If the match value is symmetrically distributed, then $u_{1}\left(v_{1}, v_{2}\right)=u_{2}\left(v_{1}, v_{2}\right)$ for any $\left(v_{1}, v_{2}\right)$.

The above lemma yields an interesting result: with the blind buying option, conditional on that she decides to sample a product in stage 1, the consumer does not care about the order in which the match values are inspected, even if the net prior values of the products are different, that is, $v_{1} \neq v_{2}$. This is in sharp contrast to the Pandora's rule of Weitzman (1979), according to which the product with a higher net prior value should always be sampled first.

The reason for Pandora's rule is that the search process without blind buying ends only when a product gets approved. Therefore, it is most efficient to sample the product with the highest net prior value first, as this product is most likely to exceed the upper reservation utility and be approved. However, with a blind buying option, another reason for the consumer to terminate the search process and make a purchase is product elimination. The consumer may want to sample the product with a lower net prior value first because it is more likely to be eliminated, in which case the consumer can stop searching and blindly buy the uninspected product. Lemma 1 states that if the match value is symmetrically distributed, the effect of product elimination offsets the effect of product approval, leaving consumers indifferent to the search order.

The intuition of Lemma 1 is as follows. ${ }^{8}$ In a duopoly market with no outside option, consumers' eventual purchase decision only depends on the difference between the payoffs generated by the two products (but not the level of the payoffs). In other words, in determining the optimal search order, what matters to the consumer is the difference of the match values $\varepsilon_{i}-\varepsilon_{j}$, where $\varepsilon_{i}$ and $\varepsilon_{j}$ are independently and identically distributed with a symmetric distribution. Then it does not matter whether $\varepsilon_{i}$ or $\varepsilon_{j}$ is revealed to the consumer, since both signals are equally informative about $\varepsilon_{i}-\varepsilon_{j} .{ }^{9}$

Finally, we return to the optimal decision of the consumer in stage 1 . According to (1), the consumer will blind buy at stage 1 if and only if $u_{0} \geq u_{1}\left(v_{1}, v_{2}\right)-c_{1}$. Our results are presented in the following proposition.

Proposition 2. Assume that the match value is symmetrical and continuously distributed and Assumption 1 holds. There exists a cutoff value $\Delta_{0}>0$, which uniquely solves

$$
\begin{equation*}
c_{1}=\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x) f\left(x+\Delta_{0}\right) d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x f\left(x+\Delta_{0}\right) d x, \tag{8}
\end{equation*}
$$

such that the optimal search strategy at stage 1 , based on the observation ( $p_{1}, p_{2}, \eta_{1}, \eta_{2}$ ), is to (a) blindly buy product 1 if $\Delta_{1} \equiv$ ( $\left.\eta_{1}-p_{1}\right)-\left(\eta_{2}-p_{2}\right)>\Delta_{0}$; (b) blindly buy product 2 if $\Delta_{1}<-\Delta_{0}$; and (c) inspect either $\tilde{\varepsilon}_{1}$ or $\tilde{\varepsilon}_{2}$ in an arbitrary sampling order if $-\Delta_{0} \leq \Delta_{1} \leq \Delta_{0}$.

Proposition 2 states that the consumer will blind buy at stage 1 if and only if the difference of the two products' net prior values, $\left|\Delta_{i}\right|$, is very large. The intuition is similar to that of Proposition 1: due to the option of "blind buying", a product will be eliminated immediately if it is too bad compared to its rival product. Sampling only takes place when the difference between the products is small.

### 2.2. Market equilibrium

We here solve firms' pricing game and focus on pure-strategy equilibria. Let $D_{i}(\mathbf{p})$ denote firm $i$ 's expected demand given any price pair $\mathbf{p}=\left(p_{1}, p_{2}\right)$, for $i \in\{1,2\}$. Let $S_{i}(\mathbf{p}, \eta)$ denote firm $i$ 's demand given price pair $\mathbf{p}$ and realizations of the prior values $\eta=\left(\eta_{1}, \eta_{2}\right)$. According to Proposition 2, we have that $S_{i}(\mathbf{p}, \eta)=0$ if $\Delta_{i}=\left(\eta_{i}-p_{i}\right)-\left(\eta_{j}-p_{j}\right)<-\Delta_{0}$, and $S_{i}(\mathbf{p}, \eta)=1$ if $\Delta_{i}>\Delta_{0}$.

Now consider the case where $-\Delta_{0} \leq \Delta_{i} \leq \Delta_{0}$. Consumers will inspect one of the products in stage 1 . Generally, firm $i$ 's demand $S_{i}(\mathbf{p}, \eta)$ may depend on whether product $i$ is first sampled. Specifically, let $q_{i}^{j}(\mathbf{p}, \eta)$ denote the probability that product $i$ is finally purchased, conditional on the consumer having inspected product $j$ 's match value in stage 1 , where $i, j$ $\in\{1,2\}$. Using Proposition 1 , we can show that both $q_{i}^{i}(\mathbf{p}, \eta)$ and $q_{i}^{j}(\mathbf{p}, \eta)$ depend only on the difference of the net prior values, $\Delta_{i}$, and satisfy that

$$
\begin{equation*}
q_{i}^{i}(\mathbf{p}, \eta)=q_{i}^{i}\left(\Delta_{i}\right) \equiv\left[1-F\left(\hat{\varepsilon}_{h}-\Delta_{i}\right)\right]+\int_{-\hat{\varepsilon}_{h}-\Delta_{i}}^{\hat{\varepsilon}_{h}-\Delta_{i}} F\left(x+\Delta_{i}\right) d F(x) \tag{9}
\end{equation*}
$$

[^4]\[

$$
\begin{equation*}
q_{i}^{j}(\mathbf{p}, \eta)=q_{i}^{j}\left(\Delta_{i}\right) \equiv F\left(\Delta_{i}-\hat{\varepsilon}_{h}\right)+\int_{\Delta_{i}-\hat{\varepsilon}_{h}}^{\Delta_{i}+\hat{\varepsilon}_{h}}\left[1-F\left(x-\Delta_{i}\right)\right] d F(x) \tag{10}
\end{equation*}
$$

\]

where $\Delta_{i} \in\left[-\Delta_{0}, \Delta_{0}\right]$.
Suppose the consumer first sampled product $i$ 's matches value $\tilde{\varepsilon}_{i}$ in stage 1 . The first term of (9), $1-F\left(\hat{\varepsilon}_{h}-\Delta_{i}\right)$, is the probability that product $i$ is purchased immediately in stage 2 (i.e., if $\Delta_{i}+\tilde{\varepsilon}_{i} \geq \hat{\varepsilon}_{h}$ ). To understand the second term, note that when $-\hat{\varepsilon}_{h}<\Delta_{i}+\tilde{\varepsilon}_{i}<\hat{\varepsilon}_{h}$ or $-\hat{\varepsilon}_{h}-\Delta_{i}<\tilde{\varepsilon}_{i}<\hat{\varepsilon}_{h}-\Delta_{i}$, the consumer continues to inspect $\tilde{\varepsilon}_{j}$ and purchase product $i$ if and only if $\Delta_{i}+\tilde{\varepsilon}_{i} \geq \tilde{\varepsilon}_{j}$. Therefore, conditional on $\tilde{\varepsilon}_{i} \in\left(-\hat{\varepsilon}_{h}-\Delta_{i}, \hat{\varepsilon}_{h}-\Delta_{i}\right)$, product $i$ is purchased with probability $F\left(\tilde{\varepsilon}_{i}+\Delta_{i}\right)$. Hence, the second term of (9) is firm $i$ 's expected demand given that both products' match values are identified. ${ }^{10}$

The explanation for (10) is similar. Suppose product $j$ is inspected first. The first term of (10) represents firm $i$ 's demand when the consumer blindly buys product $i$ after inspecting $\tilde{\varepsilon}_{j}$ and the second is firm $i$ 's expected demand when both products are inspected.

The following lemma shows that firm i's demand does not depend on the consumer's search order, so that the demand function $S_{i}(\mathbf{p}, \eta)$ is well-defined.

Lemma 2. If the match value is symmetrically distributed, then $q_{i}^{i}\left(\Delta_{i}\right)=q_{i}^{j}\left(\Delta_{i}\right)$ for any $\Delta_{i} \in\left[-\Delta_{0}, \Delta_{0}\right]$.

The above lemma states that in the case of symmetric match value, each firm receives the same demand regardless of whether its product was first sampled. This differs significantly from the typical result that firms obtain higher demand when their products are sampled earlier. The reason for our result is that, with the blind buying option, the subsequently sampled product may be bought blindly if it turns out that the first sampled product is inadequate and therefore eliminated. With a symmetric match value, the probability that a first-sampled product is accepted equals the probability that its rival product ends up being eliminated, supposing that the rival product is sampled first instead.

Therefore, given $(\mathbf{p}, \eta)$, firm $i$ 's demand is given by $S_{i}(\mathbf{p}, \eta)=S_{i}\left(\Delta_{i}\right)$, which depends only on the difference of net prior values $\Delta_{i}$ :

$$
S_{i}\left(\Delta_{i}\right)=\left\{\begin{array}{cc}
0 & \text { if } \Delta_{i}<-\Delta_{0}  \tag{11}\\
{\left[1-F\left(\hat{\varepsilon}_{h}-\Delta_{i}\right)\right]+\int_{-\hat{\varepsilon}_{h}-\Delta_{i}}^{\hat{\varepsilon}_{h}-\Delta_{i}} F\left(x+\Delta_{i}\right) d F(x)} & \text { if }-\Delta_{0} \leq \Delta_{i} \leq \Delta_{0} \\
1 & \text { if } \Delta_{i}>\Delta_{0}
\end{array}\right.
$$

We define $\delta_{i} \equiv p_{i}-p_{j}$ and $\gamma_{i} \equiv \eta_{i}-\eta_{j}$. We have $\Delta_{i}=\left(\eta_{i}-p_{i}\right)-\left(\eta_{j}-p_{j}\right)=\gamma_{i}-\delta_{i}$. It is then obvious that $\gamma_{i}$ is also symmetrically and continuously distributed. Let $K$ and $k$ be the distribution and density functions of $\gamma_{i}$, respectively. Then we have $\operatorname{Pr}\left\{\Delta_{i} \leq t\right\}=\operatorname{Pr}\left\{\gamma_{i}-\delta_{i} \leq t\right\}=K\left(t+\delta_{i}\right)$.

Hence, firm $i$ 's expected demand $D_{i}(\mathbf{p})$ is given by

$$
\begin{align*}
D_{i}(\mathbf{p}) & =\int_{-\infty}^{\infty} S_{i}(x) d K\left(x+\delta_{i}\right) \\
& =\int_{-\Delta_{0}}^{\Delta_{0}} S_{i}(x) d K\left(x+\delta_{i}\right)+1-K\left(\Delta_{0}+\delta_{i}\right) \tag{12}
\end{align*}
$$

which depends only on price difference $\delta_{i}=p_{i}-p_{j}$.
Assume the price pair $\mathbf{p}^{*}=\left(p_{1}^{*}, p_{2}^{*}\right)$ constitutes an equilibrium. Then $p_{i}^{*}$ should solve the following problem, for $i=1,2$ :

$$
\begin{equation*}
\max _{p_{i} \geq 0} p_{i} D_{i}\left(p_{i}, p_{j}^{*}\right) \tag{13}
\end{equation*}
$$

The necessary first-order condition for $p_{i}^{*}$ is that, for $i=1,2$ :

$$
\begin{equation*}
p_{i}^{*}=\frac{-D_{i}\left(\mathbf{p}^{*}\right)}{\left.\frac{\partial D_{i}\left(p_{i}, p_{j}^{*}\right)}{\partial p_{i}}\right|_{p_{i}=p_{i}^{*}}} \tag{14}
\end{equation*}
$$

Our result is summarized by the following proposition.

[^5]Proposition 3. Under Assumption 1, the pricing game has a unique symmetric pure strategy equilibrium. The equilibrium market price is as follows:

$$
\begin{equation*}
p^{*}=-\frac{\left.D_{i}\left(\delta_{i}\right)\right|_{\delta_{i}=0}}{\left.D_{i}^{\prime}\left(\delta_{i}\right)\right|_{\delta_{i}=0}}=\frac{1}{-2 k(0)+4\left[k\left(\Delta_{0}\right)-\int_{0}^{\Delta_{0}} S_{i}(x) k^{\prime}(x) d x\right]} \tag{15}
\end{equation*}
$$

When density functions $f$ and $g$ are log-concave, both $S_{i}$ and $K$ are log-concave, which implies demand function $D_{i}(\mathbf{p})$ is also log-concave in $p_{i}$. The log concavity of the demand function guarantees that: (i) the necessary (first-order) conditions (14) suffice to characterize the solution of the maximization problem (13) and (ii) a solution to the equation system (14) exists. These conditions give the existence of the pricing equilibrium. Finally, the uniqueness of the pricing equilibrium stems from the property that demand function $D_{i}(\mathbf{p})$ only depends on price difference $\delta_{i}=p_{i}-p_{j}$ and is log-concave in $\delta_{i}$.

## 3. Comparison

This section examines the comparative statistics regarding the effects of blind buying. We first examine how the presence of a blind buying option affects the equilibrium price. Then we investigate the relationship between market prices and search costs with the blind buying option. We finally examine the effects of blind buying on welfare.

We first review the case without the blind buying option (e.g., Choi et al., 2018; Haan et al., 2018) in a symmetric duopoly setting, in which the match value is continuously and symmetrically distributed.

Suppose $c_{1}$ and $c_{2}$ are the search costs for the first and second samples, respectively. We remain to assume that each consumer's outside option is $-\infty$ so that she always inspects a product in stage 1 . Obviously, without "blind buying", the first-sample search cost $c_{1}$ has no effect on the market equilibrium. The optimal search strategy depends only on the second-sample search $\operatorname{cost} c_{2}$ and is fully characterized by Pandora's rule. ${ }^{11}$

Recall that $\Delta_{i}=\left(\eta_{i}-p_{i}\right)-\left(\eta_{j}-p_{j}\right)$ is the difference between the net prior values for $i=1,2$. Then the consumer will first sample product $i$ (or product $j$ ) if $\Delta_{i}>0$ (or $\Delta_{i}<0$ ). The sampled product is purchased immediately if the match value is sufficiently high. Otherwise the consumer continues the search. The optimal search strategy thus is summarized by the following lemma.

Lemma 3. Assume blind buying is not allowed and Assumption 1 holds. The consumer always first samples product $i$, which satisfies $\Delta_{i} \geq 0$. After observing $\varepsilon_{i}$, the consumer (a) continues to inspect $\tilde{\varepsilon}_{j}$ if $\Delta_{i}+\varepsilon_{i}<\hat{\varepsilon}_{h}$ and (b) stops searching and purchases product $i$ if $\Delta_{i}+\varepsilon_{i} \geq \hat{\varepsilon}_{h}$.

Let $S_{i}^{n}(\mathbf{p}, \eta)$ denote firm $i$ 's demand given $(\mathbf{p}, \eta)$, for $i=1,2$. Throughout this section, we use superscript $n$ to denote the corresponding variables for the case without a blind buying option. Then, according to the optimal search strategy, the demand function $S_{i}^{n}(\mathbf{p}, \eta)$ depends only on $\Delta_{i}$ and is given as follows:

$$
S_{i}^{n}(\mathbf{p}, \eta)=S_{i}^{n}\left(\Delta_{i}\right)=\left\{\begin{array}{cl}
{\left[1-F\left(\hat{\varepsilon}_{h}-\Delta_{i}\right)\right]+\int_{-\infty}^{\hat{\varepsilon}_{h}-\Delta_{i}} F\left(x+\Delta_{i}\right) d F(x)} & \text { if } \Delta_{i}>0 \\
\int_{-\infty}^{\Delta_{i}+\hat{\varepsilon}_{h}}\left[1-F\left(x-\Delta_{i}\right)\right] d F(x) & \text { if } \Delta_{i}<0
\end{array} .\right.
$$

Note that, unlike $S_{i}(\cdot)$, the above demand function $S_{i}^{n}(\cdot)$ is discontinuous at $\Delta_{i}=0$. Specifically, it is easy to calculate that

$$
\begin{aligned}
\lim _{\Delta_{i} \rightarrow 0+} S_{i}^{n}\left(\Delta_{i}\right) & =1-F\left(\hat{\varepsilon}_{h}\right)+\frac{1}{2} F\left(\hat{\varepsilon}_{h}\right)^{2} \\
\lim _{\Delta_{i} \rightarrow 0-} S_{i}^{n}\left(\Delta_{i}\right) & =F\left(\hat{\varepsilon}_{h}\right)-\frac{1}{2} F\left(\hat{\varepsilon}_{h}\right)^{2} .
\end{aligned}
$$

The gap between $\lim _{\Delta_{i} \rightarrow 0+} S_{i}^{n}\left(\Delta_{i}\right)$ and $\lim _{\Delta_{i} \rightarrow 0-} S_{i}^{n}\left(\Delta_{i}\right)$, which equals $\left[1-F\left(\hat{\varepsilon}_{h}\right)\right]^{2}>0$, characterizes the value of being prominent, as widely noted in traditional search models without a blind buying option. It is thus obvious that the advantage of prominence decreases with the reservation value $\hat{\varepsilon}_{h}$ and increases with the second-sample search cost $c_{2}$.

Similar to the previous analysis, firm $i$ 's expected demand depends only on price difference $\delta_{i}=p_{i}-p_{j}$ and is given by

$$
D_{i}^{n}(\mathbf{p})=D_{i}^{n}\left(\delta_{i}\right)=\int_{-\infty}^{\infty} S_{i}^{n}(x) d K\left(x+\delta_{i}\right)
$$

[^6]Finally, the unique price equilibrium for the case without a blind buying option is as follows:

$$
\begin{equation*}
p_{n}^{*}=-\frac{D_{i}^{n}(\delta=0)}{D_{i}^{n \prime}(\delta=0)}=-\frac{1}{2 k(0)+4 \int_{0}^{\infty} S_{i}^{n}(x) k^{\prime}(x) d x} \tag{16}
\end{equation*}
$$

### 3.1. The effects of blind buying on equilibrium prices

We first examine how the presence of a blind buying option affects the equilibrium price. Our result is presented in the following proposition.

Proposition 4. Under Assumption 1, there exists a cutoff value $\hat{c}_{1}$ such that, $p^{*}<p_{n}^{*}$ if and only if $c_{1}>\hat{c}_{1}$. That is, allowing blind buying decreases (increases) market price if the first-sample search cost is high (low).

Intuitively, in price-directed search models without the blind buying option, search frictions have two effects on price competition. First, higher search costs make consumers more hesitate to inspect the next product. Search frictions cause a customer-retention effect in the sense that an earlier-visited firm can retain the consumer and induce an immediate purchase as long as its net product value exceeds the reservation utility. The customer-retention effect reduces price competition because search frictions generate monopoly power to the earlier-visited firms. Second, due to the customer-retention effect and Weitzman's optimal search policy, when prices are observable, firms are incentivized to lower product prices in order to be earlier visited. This effect increases price competition and is called the early-visit effect.

The introduction of a blind buying option brings two important changes. First, blind buying eliminates the early-visit effect. The reason is twofold. On the one hand, consumers are indifferent to the search order, conditional on sampling in stage 1. This means that lowering prices cannot increase firms' probabilities of being sampled earlier. On the other hand, firms' profits are independent of consumers' search order. Thus, firms have no incentive to lower their prices in order to attract first visits.

Second, blind buying creates a new effect on price competition, which we call the blind-buying effect, in the sense that an currently visited firm may lose its consumer if its product is too "bad", i.e., $\Delta_{i}=\left(\eta_{i}-p_{i}\right)-\left(\eta_{j}-p_{j}\right)<-\Delta_{0}$ in stage 1 , or $\Delta_{i}+\varepsilon_{i}<-\hat{\varepsilon}_{h}$ in stage 2 , in which cases the consumer will blindly buy its rival product. The threat of blind buyings (from the rivals) induces firms to increase their product values through lower prices, which increases price competition.

Specifically, the blind-buying effect exists in both stages. According to the consumer's optimal search behavior, the firststage blind-buying effect is characterized by $\Delta_{0}$, the width of the net prior value difference, which is given by Equation (8):

$$
c_{1}=\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x) f\left(x+\Delta_{0}\right) d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x f\left(x+\Delta_{0}\right) d x
$$

where $H(x)=-c_{2}+x+\int_{x}^{\infty}(t-x) d F(t)$.
The second-stage blind-buying effect is characterized by the lower reservation utility, $-\hat{\varepsilon}_{h}$, which is given by Equation (6):

$$
c_{2}=\int_{\hat{\varepsilon}_{h}}^{\infty}\left(t-\hat{\varepsilon}_{h}\right) d F(t)
$$

A smaller $\Delta_{0}\left(\hat{\varepsilon}_{h}\right)$ means a greater threat that the consumer will blindly buy the other (uninspected) product in the first stage (in the second stage), and thus leads to a higher first-stage blind-buying effect (second-stage blind-buying effect). Since $\Delta_{0}$ decreases with the first-sample search cost $c_{1}$, a higher $c_{1}$ increases the first-stage blind-buying effect. In other words, a higher $c_{1}$ makes the consumer more hesitate to sample the first product and thus increases the probability of blind buying in stage 1 .

The intuition for Proposition 4 is as follows. The impact of blind buying on equilibrium prices is twofold. On the one hand, a firm has no incentive to lower its price in order to attract first visits. On the other hand, the option of blind buying (in stage 1) allows consumers to make uninformed purchasing decisions, i.e., purchase without inspecting any products. That consumers possess less information about their tastes reduces products' heterogeneity and increases price competition. ${ }^{12}$

In other words, the presence of blind buying replaces the early-visit effect with the blind-buying effect. Since both effects intensify price competition, the presence of blind buying lowers equilibrium prices if and only if the blind-buying effect dominates the early-visit effect. Note that an increase in the first-sample search cost $c_{1}$ has no impact on the early-visit effect, ${ }^{13}$ but increases the first-stage blind-buying effect. Thus, if $c_{1}$ is sufficiently high, i.e., above some cut-off value $\hat{c}_{1}$, the

[^7]blind-buying effect dominates the early-visit effect so that the option of blind buying increases price competition, resulting in a lower market price.

### 3.2. The effects of higher search costs

We now consider the effect of an increase in search costs on market prices when blind buying is available.

Proposition 5. When blind buying is allowed, under Assumption 1, the equilibrium price decreases with both $c_{1}$ and $c_{2}$.
As discussed in Section 3.1, the presence of blind buying replaces the early-visit effect with a blind-buying effect. To see how market prices change with the search costs, we need to compare the customer-retention effect with the blind-buying effect.

From Equation (8), an increase in the first-sample search cost $c_{1}$ lowers $\Delta_{0}$, which strengthens the first-stage blind-buying effect, but affects neither the second-stage blind-buying effect nor the customer-retention effect, since $\hat{\varepsilon}_{h}$ does not change with $c_{1}$. Thus, a higher $c_{1}$ always increases price competition and lowers market prices. This result is in sharp contrast to the ones without blind buying, where the first-sample search cost has no effect on equilibrium outcomes.

For the second-sample search $\operatorname{cost} c_{2}$, similar to the case without blind buying, a higher $c_{2}$ increases the customerretention effect, which weakens price competition due to the monopoly power of the first-sampled firm. On the other hand, due to Equation (6), since both $\Delta_{0}$ and $\hat{\varepsilon}_{h}$ decrease with $c_{2}$, an increase in $c_{2}$ strengthens both the first-stage and the second-stage blind-buying effects, intensifying price competition. Proposition 5 states that the overall blind-buying effect always dominates the customer-retention effect so that the market price decreases with $c_{2}$.

In fact, we can show that the second-stage blind-buying effect suffices to dominate the customer-retention effect. Consider the extreme case where $c_{1} \rightarrow 0$ so that $\Delta_{0} \rightarrow \infty$. In this case, the first-stage blind-buying effect disappears since blind buying never takes place in the first stage. Thus, an increase in $c_{2}$ strengthens only the second-stage blind-buying effect. Our result that the equilibrium price always decreases with $c_{2}$ implies that the customer-retention effect is actually dominated by the second-stage blind-buying effect.

Although the effect of the second-sample search cost on market prices is the same as in existing price-directed search models (e.g., Armstrong and Zhou, 2011; Choi et al., 2018; Haan et al., 2018; Shen, 2016), the underlying mechanisms are different. In existing price-directed search models, since prices are observable, a price cut can attract more consumers to visit a firm, and these first visits are more likely to lead to final sales as search costs increase. As a result, firms' incentives to lower their prices increase with the search costs, resulting in a negative relationship between equilibrium prices and search costs. In other words, without blind buying, it is the early-visit effect that intensifies price competition and leads to lower market prices. However, with the option of blind buying, the early-visit effect does not exist. What increases price competition is the new-created blind-buying effect. With blind buying, firms lower prices not for being sampled earlier, but for higher product values so that consumers are less likely to blindly buy from their rivals.

### 3.3. Welfare comparison

In this subsection, we study the effects of blind buying on industry profit, consumer surplus and total welfare. Note that the equilibrium profit for each firm is given by $\pi^{*}=\frac{1}{2} p^{*}$ (or $\pi_{n}^{*}=\frac{1}{2} p_{n}^{*}$ ) if blind buying is allowed (not allowed). The comparison of industry profits follows immediately from Proposition 4.

To compare consumer surplus and total welfare, let $E U^{b}\left(p_{1}, p_{2}\right)$ and $E U^{n}\left(p_{1}, p_{2}\right)$ denote consumers' expected utility under price vector $\left(p_{1}, p_{2}\right)$ for the case with and without blind buying, respectively. Since the consumer's optimal search strategy depends only on the price difference, for any $t \in\{b, n\}$ and $p>0$ :

$$
E U^{t}(p, p)=E U^{t}(0,0)-p
$$

Thus, consumer surplus $C S^{t}$ and total welfare $T W^{t}$ for the two models, $t \in\{b, n\}$, are as follows:

$$
\begin{aligned}
C S^{b} & =E U^{b}\left(p^{*}, p^{*}\right)=E U^{b}(0,0)-p^{*} \\
C S^{n} & =E U^{n}\left(p_{n}^{*}, p_{n}^{*}\right)=E U^{n}(0,0)-p_{n}^{*} \\
T W^{b} & =E U^{b}(0,0) \\
T W^{n} & =E U^{n}(0,0)
\end{aligned}
$$

Our results are summarized in the following proposition.
Proposition 6. When blind buying is allowed, under Assumption 1, (i) industry profit increases if and only if the first-sample search $\operatorname{cost} c_{1}$ is below some cutoff value $\hat{c}_{1}$; (ii) consumer surplus unambiguously increases when the first-sample search cost $c_{1}$ is high, and can either increase or decrease when $c_{1}$ is low; (iii) total welfare always increases.

That the availability of blind buying increases total welfare comes from the fact that $E U^{b}(0,0)>E U^{n}(0,0)$, which is intuitive. When products are free, the consumer is always better off with a blind buying option. The benefits are achieved when the sampled product has a low realized match value, in which case blindly buying the rival product saves the search cost and gives the consumer a higher expected utility.

The effect of blind buying on consumer surplus depends on the first-sample search cost. When $c_{1}$ is high, blind buying lowers market prices and industry profit, thus increasing consumer surplus. When $c_{1}$ is small, while blind buying increases total welfare, firms also enjoy higher expected profits. Thus, whether consumer surplus increases depends on the comparison between the benefits of the firms and the welfare increase, which is determined by the shapes of the primitive distributions.

## 4. The curse of prominence

Here, we demonstrate that, with blind buying, prominence could be harmful to a firm, which is in contrast to the conventional wisdom that prominence is always beneficial (Armstrong et al., 2009; Armstrong and Zhou, 2011). To this end, we need to assume that the distribution of the match value is asymmetric. To ensure tractability, we consider the case of a binary match value. That is, we assume $\tilde{\varepsilon}_{i} \in\{0, r\}$, with $\operatorname{Pr}\left(\tilde{\varepsilon}_{i}=r\right)=\alpha$.

### 4.1. Exogenous prominence

This subsection examines the case of exogenous prominence. Assume that firm 1 is the prominent firm and firm 2 is the non-prominent firm. Following Armstrong et al. (2009), we assume that all consumers first visit the prominent firm at no cost, and then choose whether to visit the non-prominent firm at cost $c_{2}$. To provide a justification for such an assumption, consider the online-and-offline example that we give in the introduction. If firm 1 opens its offline store in a convenient location and firm 2 opens its offline store in an unfavorable location, we can assume that all consumers will first visit the firm 1's offline store.

When blind buying is not allowed, it can be shown that being prominent is beneficial. ${ }^{14}$ Now we show that prominence can be harmful when blind buying is allowed. With observation ( $p_{1}, p_{2}, \eta_{1}, \eta_{2}, \varepsilon_{1}$ ), the consumer has three options: 1 ) buys product 1 without further inspection and obtains $v_{1}+\varepsilon_{1},{ }^{15} 2$ ) blindly buys product 2 and obtains $v_{2}+E \tilde{\varepsilon}_{2}=v_{2}+\alpha r$, or 3 ) inspects product 2 .

If $v_{1}+\varepsilon_{1} \geq v_{2}+\alpha r$, the consumers choose between 1$)$ and 3 ). The value of inspecting $\tilde{\varepsilon}_{2}$ is $\alpha\left(v_{2}+r-\left(v_{1}+\varepsilon_{1}\right)\right.$ ), and the consumer chooses to inspect $\tilde{\varepsilon}_{2}$ if $\alpha\left(v_{2}+r-\left(v_{1}+\varepsilon_{1}\right)\right)>c_{2}$, which can be rewritten as:

$$
\begin{equation*}
\Delta_{1}+\varepsilon_{1}<r-\frac{c_{2}}{\alpha} \tag{17}
\end{equation*}
$$

where $\Delta_{1}=v_{1}-v_{2}$.
If $v_{1}+\varepsilon_{1}<v_{2}+\alpha r$, the consumer chooses between 2$)$ and 3 ). The value of inspecting $\tilde{\varepsilon}_{2}$ is $(1-\alpha)\left(v_{1}+\varepsilon_{1}-v_{2}\right)$ and the consumer chooses to inspect $\tilde{\varepsilon}_{2}$ if $(1-\alpha)\left(v_{1}+\varepsilon_{1}-v_{2}\right)>c_{2}$, which can be rewritten as:

$$
\begin{equation*}
\Delta_{1}+\varepsilon_{1}>\frac{c_{2}}{1-\alpha} \tag{18}
\end{equation*}
$$

To ensure the search takes place, we assume the search cost is not too high.
Assumption 2. $c_{2}<\alpha(1-\alpha) r$.
Under Assumption 2, our analysis above yields the following lemma:
Lemma 4. If Assumption 2 holds, a consumer who has sampled firm 1 will
a) buy product 1 without further inspection if $\Delta_{1}+\varepsilon_{1} \geq r-\frac{c_{2}}{\alpha}$;
b) blindly buy product 2 if $\Delta_{1}+\varepsilon_{1}<\frac{c_{2}}{1-\alpha}$;
c) continue to inspect product 2 if $\frac{c_{2}}{1-\alpha} \leq \Delta_{1}+\varepsilon_{1}<r-\frac{c_{2}}{\alpha}$; then buy product 2 if $\varepsilon_{2}=r$ or buy product 1 if $\varepsilon_{2}=0$.

The consumer will continue to search only if the current option $\Delta_{1}+\varepsilon_{1}$ is neither too high nor too low. Otherwise, the consumer either buys product 1 immediately or buys product 2 blindly.

Now we calculate each firm $i$ 's expected demand $D_{i}(\mathbf{p})$. Because we assume that all consumers make final purchases, we have $D_{1}(\mathbf{p})=1-D_{2}(\mathbf{p})$. Therefore, it suffices to calculate only one firm's demand and we focus on $D_{2}(\mathbf{p})$. We denote $D_{2}\left(\mathbf{p} \mid \varepsilon_{1}\right)$ as firm 2's demand conditional on $\varepsilon_{1}$; then we have $D_{2}(\mathbf{p})=\alpha D_{2}\left(\mathbf{p} \mid \varepsilon_{1}=r\right)+(1-\alpha) D_{2}\left(\mathbf{p} \mid \varepsilon_{1}=0\right)$. Recall that $\delta_{1}=p_{1}-p_{2}, \gamma_{1}=\eta_{1}-\eta_{2}, \Delta_{1}=\gamma_{1}-\delta_{1}$ and that $\gamma_{1}$ has c.d.f. $K$ and p.d.f. $k$. Then, according to the above lemma, we have

[^8]\[

$$
\begin{aligned}
D_{2}\left(\mathbf{p} \mid \varepsilon_{1}\right) & =\operatorname{Pr}\left(\Delta_{1}+\varepsilon_{1}<\frac{c_{2}}{1-\alpha}\right)+\alpha \operatorname{Pr}\left(\frac{c_{2}}{1-\alpha} \leq \Delta_{1}+\varepsilon_{1}<r-\frac{c_{2}}{\alpha}\right) \\
& =K\left(\frac{c_{2}}{1-\alpha}-\varepsilon_{1}+\delta_{1}\right)+\alpha\left[K\left(r-\frac{c_{2}}{\alpha}-\varepsilon_{1}+\delta_{1}\right)-K\left(\frac{c_{2}}{1-\alpha}-\varepsilon_{1}+\delta_{1}\right)\right]
\end{aligned}
$$
\]

where the first term on the right-hand side is the probability of product 2 being blindly bought, and the second term is the purchasing probability the consumer continues to sample product 2 and observes a good match value.

Firm 2's expected demand is hence given by

$$
\begin{align*}
D_{2}(\mathbf{p}) & =\alpha\left[(1-\alpha) K\left(\frac{c_{2}}{1-\alpha}-r+\delta_{1}\right)+\alpha K\left(\delta_{1}-\frac{c_{2}}{\alpha}\right)\right]  \tag{19}\\
& +(1-\alpha)\left[(1-\alpha) K\left(\frac{c_{2}}{1-\alpha}+\delta_{1}\right)+\alpha K\left(r-\frac{c_{2}}{\alpha}+\delta_{1}\right)\right]
\end{align*}
$$

which only depends on the price difference $\delta_{1}$ and is increasing in $\delta_{1}$.

Lemma 5. Suppose Assumption 2 holds. If $\delta_{1}=0$, we have $D_{2}(\mathbf{p}) \geq D_{1}(\mathbf{p})$ if and only if $\alpha \leq 1 / 2$.

If $\delta_{1}=0$, both firms charge the same prices. Firms 1 and 2 are thus identical except that firm 1 has a prominent position. The lemma therefore suggests that being prominent could reduce a firm's demand, which is contrary to the conventional wisdom that a prominent position generates higher demand. There are pros and cons of being a prominent firm in the presence of blind buying. On the one hand, the prominent firm has a high chance of being bought immediately if its unobservable match value turns out to be of high value. On the other hand, the non-prominent firm has a high chance of being bought blindly if the unobservable match value of the prominent firm turns out to be of low value. If the unobservable match value is more likely to be of low value ( $\alpha<\frac{1}{2}$ ), then being non prominent can be beneficial. The intuition is clearest in the case when the two firms' net prior values are the same ( $v_{1}=v_{2}$ ): Then, the consumer will buy from firm 1 if $\varepsilon_{1}=r$ and blindly buy from the firm 2 if $\varepsilon_{1}=0$. Obviously, when $\alpha<1 / 2$, the non-prominent firm enjoys a higher demand.

Since prominence can be both an advantage and a disadvantage to generating demand, it also has an ambiguous effect on the price and profit of a firm, as the following proposition confirms.

Proposition 7. In equilibrium, the prominent firm charges a lower (higher) price and obtains a lower (higher) profit than the nonprominent firm if $\alpha \leq 1 / 2(\alpha \geq 1 / 2)$.

When $\alpha \leq 1 / 2$, a consumer has a high likelihood of buying blindly from the non-prominent firm, thus the prominent firm has strong incentive to lower its price to generate high demand. In contrast, its non-prominent rival has a weak incentive to lower its price because it enjoys high demand from consumers' blind buying. As a result, the prominent firm charges a lower price than its non-prominent competitor. When $\alpha \geq 1 / 2$, the prominent firm benefits from the high probability of being bought immediately and therefore, has weak incentive to lower its price. However, its non-prominent rival is actively trying to attract more demand through price cuts. As a result, the prominent firm charges a higher price than its non-prominent competitor.

There are two differences between our results and those of Armstrong et al. (2009). First, we suggest that being prominent can be harmful to a firm's profit, which contrary to Armstrong et al. (2009). Intuitively, with a blind buying option, being non-prominent has the advantage of being bought blindly and thus may increase one's profit. Second, if prominence is beneficial for a firm ( $\alpha \geq 1 / 2$ ) as in Armstrong et al. (2009), the prediction on market prices differs. According to Armstrong et al. (2009), the prominent firm charges a lower price than its non-prominent competitors and generates a higher profit due to higher demand. By contrast, in our model, the price of the prominent firm is higher than that of its non-prominent competitor, which is in line with the results of Arbatskaya (2007). While Arbatskaya (2007) considers homogeneous products, our results show that her results can apply in case of horizontal differentiation.

From the perspective of information disclosure, our result exists some similarities as Lewis and Sappington (1994). In their paper, the authors study a monopolist's decision of whether to disclose information to consumers about their tastes. Supplying information to consumers enables a monopolist to segment the market and charge higher price to high-value consumers, but it also increases consumers' information rent. The monopolist may be better by withholding information. In our paper here, the prominent firm is essentially forced to reveal its information, and may do worse than the other firm, for whom being bought blindly is still an option.

### 4.2. Endogenous prominence

This subsection examines the case when consumers can endogenously choose the search order based on the information they have observed. Prominence is therefore endogenous and depends on firms' price strategies. This subsection also com-
plements our basic model by considering an asymmetric distribution of the match value. ${ }^{16}$ Denote $c_{n}$ as the search cost of a consumer's $n$-th sample as in our basic model, for $n=1,2$.

In stage 1, a consumer decides among four options: 1) Search product 1 first; 2) Search product 2 first; 3) Blindly buy product 1 without any search; 4) Blindly buy product 2 without any search.

Suppose a consumer search product $i$ first, then her subsequent decisions are given by Lemma 4. Accordingly, one can easily derive $u_{i}\left(v_{1}, v_{2}\right)$, which is the consumer's expected utility when product $i$ is first sampled. On the other hand, if the consumer blindly buys product $i$, her expected utility is equal to $v_{i}+\alpha r$. Comparing $v_{i}+\alpha r, v_{j}+\alpha r, u_{i}\left(v_{1}, v_{2}\right)-c_{1}$ and $u_{j}\left(v_{1}, v_{2}\right)-c_{1}$, we can solve the optimal search order for the consumer.

Assumption 3. $c_{1}+c_{2}<\alpha(1-\alpha) r$.

Lemma 6. Suppose Assumption 3 holds.
i) If $\left|\Delta_{i}\right| \geq r-\frac{c_{1}+\alpha c_{2}}{\alpha(1-\alpha)}$, then the consumers blindly buy the product with the higher net prior value (i.e., higher $v_{i}$ ).
ii) If $\left|\Delta_{i}\right|<r-\frac{c_{1}+\alpha c_{2}}{\alpha(1-\alpha)}$, then the consumer searches the product with a lower (resp. higher) net prior value if $\alpha<1 / 2$ (resp. $\alpha \geq 1 / 2)$.

If the difference between the net prior values of the two products is sufficiently large, the consumer will not search and blindly buy the product with the higher net prior value. Otherwise, the consumer will search. Interestingly, if the match value is more likely to be low ( $\alpha<1 / 2$ ), the consumer first inspects the product with a lower net prior value. Intuitively, by inspecting the product with a lower net prior value, the consumer is more likely to end the search by eliminating it, thereby saving on search costs. Armstrong and Zhou (2011) suggest different types of being prominent. One of them is lowering the product price. By contrast, our lemma above suggests that a firm can gain search prominence ${ }^{17}$ by increasing its product price. However, be aware that prominence is harmful to the firm when $\alpha<1 / 2$. A firm therefore has the incentive to avoid being prominent by lowering its price.

Based on the search behavior of consumers, it is easy to derive the demand function of each firm. Once we get the demand functions, we can solve the equilibrium prices based on the first order condition of firms' optimization problem and then conduct comparative statistics.

Proposition 8. Suppose $c_{2}$ is sufficiently small. Then, an increase in $c_{1}$ or $c_{2}$ reduces the equilibrium prices.

As $c_{1}$ increases, firms lower their prices to attract more first-stage blind buying. The impact of an increase in $c_{2}$ on prices is the same as with existing price-directed search models. However, the underlying mechanisms are quite different. As $c_{2}$ increases, firms lower their prices to avoid being sampled first in our model if $\alpha<1 / 2$, while this is happening to get more first sampling in existing price-directed search models.

## 5. Conclusions

This paper analyzes a price-directed search model in a duopoly market, in which consumers can buy a product without inspecting it. We show that Weitzman's (1979) optimal search policy no longer applies. When the match value has a symmetric distribution, both consumers and firms are indifferent to the search order, conditional on searching. While an increase in the first-sample search cost has no effect on market prices in existing price-directed search models, it reduces equilibrium prices in our model. The effect of second-sample search cost on market prices is the same as in existing pricedirected search models, but the underlying mechanisms are quite different. We also show that, when the match value has asymmetric distribution, being prominent can be harmful to a firm.

Welfare analysis further shows that, blind buying increases market prices and industrial profits if and only if the firstsample search cost is below a threshold value, while it always increases total welfare. Consequently, blind buying increases consumer surplus if first-sample search cost is high, but has an ambiguous effect on consumer surplus if the first-sample search cost is low.

However, there are many questions that remain unanswered by our results. For instance, our model only considers a duopoly market. It is thus natural to extend our model to an oligopoly market with more than two firms. We are aware that such an extension poses significant technical challenges. The main difficult is that there is no general and tractable solution for consumers' optimal search policy. Particularly, the search order may be history dependent (Doval, 2018). Nonetheless, overcoming these difficulties has a high theoretical value and requires further research. For an asymmetric match value, we only consider a binary distribution. Studying a continuous asymmetric distribution is theoretically interesting, but extremely challenging.

[^9]
## 6. Appendix

Proof of Proposition 1. Proposition 1 follows immediately from (6) and (7) and the fact that $H^{\prime}(x)=F(x) \in(0,1)$.
Proof of Lemma 1. Note that $u_{1}\left(v_{1}, v_{2}\right)$ and $u_{2}\left(v_{1}, v_{2}\right)$ are given as follows.

$$
\begin{aligned}
& u_{1}\left(v_{1}, v_{2}\right)=\int_{-\infty}^{-\hat{\varepsilon}_{h}-\Delta_{1}} v_{2} d F(t)+\int_{-\hat{\varepsilon}_{h}-\Delta_{1}}^{\hat{\varepsilon}_{h}-\Delta_{1}}\left[H\left(\Delta_{1}+t\right)+v_{2}\right] d F(t)+\int_{\hat{\varepsilon}_{h}-\Delta_{1}}^{\infty}\left(v_{1}+t\right) d F(t) \\
& u_{2}\left(v_{1}, v_{2}\right)=\int_{-\infty}^{-\hat{\varepsilon}_{h}+\Delta_{1}} v_{1} d F(t)+\int_{-\hat{\varepsilon}_{h}+\Delta_{1}}^{\hat{\varepsilon}_{h}+\Delta_{1}}\left[H\left(-\Delta_{1}+t\right)+v_{1}\right] d F(t)+\int_{\hat{\varepsilon}_{h}+\Delta_{1}}^{\infty}\left(v_{2}+t\right) d F(t)
\end{aligned}
$$

where $\Delta_{1}=v_{1}-v_{2}$. The above expression for $u_{1}\left(v_{1}, v_{2}\right)$ can be rewritten as

$$
\begin{aligned}
u_{1}\left(v_{1}, v_{2}\right) & =v_{2}+\int_{-\hat{\varepsilon}_{h}-\Delta_{1}}^{\hat{\varepsilon}_{h}-\Delta_{1}} H\left(\Delta_{1}+t\right) f(t) d t+\int_{\hat{\varepsilon}_{h}-\Delta_{1}}^{\infty}\left(\Delta_{1}+t\right) f(t) d t \\
& =v_{2}+\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x) f\left(x-\Delta_{1}\right) d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x f\left(x-\Delta_{1}\right) d x
\end{aligned}
$$

Similarly, $u_{2}\left(v_{1}, v_{2}\right)$ can be rewritten as:

$$
u_{2}\left(v_{1}, v_{2}\right)=v_{1}+\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x) f\left(x+\Delta_{1}\right) d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x f\left(x+\Delta_{1}\right) d x
$$

from which we have

$$
\begin{equation*}
u_{1}\left(v_{1}, v_{2}\right)-u_{2}\left(v_{1}, v_{2}\right)=-\Delta_{1}+\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x)\left[f\left(x-\Delta_{1}\right)-f\left(x+\Delta_{1}\right)\right] d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x\left[f\left(x-\Delta_{1}\right)-f\left(x+\Delta_{1}\right)\right] d x \tag{20}
\end{equation*}
$$

Since $H^{\prime}=F$ and $F(x)=1-F(-x)$, it is easy to show that

$$
\begin{equation*}
H(x)-H(-x)=x, \text { for any } x \tag{21}
\end{equation*}
$$

Using $y=-x$ and applying integration by substitution, we have

$$
\begin{aligned}
\int_{-\hat{\varepsilon}_{h}}^{0} H(x)\left[f\left(x-\Delta_{1}\right)-f\left(x+\Delta_{1}\right)\right] d x & =\int_{0}^{\hat{\varepsilon}_{h}} H(-y)\left[f\left(-y-\Delta_{1}\right)-f\left(-y+\Delta_{1}\right)\right] d y \\
& =\int_{0}^{\hat{\varepsilon}_{h}}[y-H(y)]\left[f\left(y-\Delta_{1}\right)-f\left(y+\Delta_{1}\right)\right] d y
\end{aligned}
$$

where we use equation (21) and the symmetry of $f$.
Therefore, expression (20) can be rewritten as

$$
\begin{aligned}
u_{1}\left(v_{1}, v_{2}\right)-u_{2}\left(v_{1}, v_{2}\right) & =-\Delta_{1}+\int_{0}^{\hat{\varepsilon}_{h}} x\left[f\left(x-\Delta_{1}\right)-f\left(x+\Delta_{1}\right)\right] d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x\left[f\left(x-\Delta_{1}\right)-f\left(x+\Delta_{1}\right)\right] d x \\
& =-\Delta_{1}+\int_{0}^{\infty} x\left[f\left(x-\Delta_{1}\right)-f\left(x+\Delta_{1}\right)\right] d x
\end{aligned}
$$

$$
\begin{aligned}
& =-\Delta_{1}+\frac{1}{2} \int_{-\infty}^{\infty} x\left[f\left(x-\Delta_{1}\right)-f\left(x+\Delta_{1}\right)\right] d x \\
& =-\Delta_{1}+\frac{1}{2}\left[\int_{-\infty}^{\infty}\left(t+\Delta_{1}\right) f(t) d t-\int_{-\infty}^{\infty}\left(t-\Delta_{1}\right) f(t) d t\right] \\
& =-\Delta_{1}+\frac{1}{2} \int_{-\infty}^{\infty} 2 \Delta_{1} f(t) d t=0
\end{aligned}
$$

where the third equation follows from $x\left[f\left(x-\Delta_{1}\right)-f\left(x+\Delta_{1}\right)\right]$ being symmetric in $x$ and the fourth equation is a result of integration by substitution.

Proof of Proposition 2. The expected benefit of sampling a product in stage 1 is $B\left(v_{1}, v_{2}\right)=u_{1}\left(v_{1}, v_{2}\right)-\max \left(v_{1}, v_{2}\right)=$ $u_{2}\left(v_{1}, v_{2}\right)-\max \left(v_{1}, v_{2}\right)$. A blind buying takes place in stage 1 if and only if $B\left(v_{1}, v_{2}\right)<c_{1}$.

From the proof of Lemma 1, we have

$$
\begin{aligned}
& u_{1}\left(v_{1}, v_{2}\right)=v_{2}+\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x) f\left(x-\Delta_{1}\right) d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x f\left(x-\Delta_{1}\right) d x \\
& u_{2}\left(v_{1}, v_{2}\right)=v_{1}+\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x) f\left(x+\Delta_{1}\right) d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x f\left(x+\Delta_{1}\right) d x
\end{aligned}
$$

where $\Delta_{1}=v_{1}-v_{2}$. Thus, the expected benefit $B\left(v_{1}, v_{2}\right)$ depends only on the net prior value difference $\Delta_{1}$ and is given as follows

$$
\begin{aligned}
B\left(v_{1}, v_{2}\right) & =B\left(\Delta_{1}\right) \\
& =\left\{\begin{array}{ll}
\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x) f\left(x+\Delta_{1}\right) d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x f\left(x+\Delta_{1}\right) d x & \text { if } \Delta_{1} \geq 0, \text { i.e., } v_{1} \geq v_{2} \\
\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x) f\left(x-\Delta_{1}\right) d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x f\left(x-\Delta_{1}\right) d x & \text { if } \Delta_{1}<0, \text { i.e., } v_{1}<v_{2}
\end{array} .\right.
\end{aligned}
$$

The above function $B\left(\Delta_{1}\right)$ is symmetric in $\Delta_{1}$. Without loss of generality, we focus on the case where $\Delta_{1} \geq 0$, so that

$$
B\left(\Delta_{1}\right)=\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x) f\left(x+\Delta_{1}\right) d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x f\left(x+\Delta_{1}\right) d x
$$

Using $t=x+\Delta_{1}$, we can rewrite the above expression as follows

$$
B\left(\Delta_{1}\right)=\int_{-\hat{\varepsilon}_{h}+\Delta_{1}}^{\hat{\varepsilon}_{h}+\Delta_{1}} H\left(t-\Delta_{1}\right) f(t) d t+\int_{\hat{\varepsilon}_{h}+\Delta_{1}}^{\infty}\left(t-\Delta_{1}\right) f(t) d t
$$

Thus,

$$
\begin{aligned}
\frac{\partial B\left(\Delta_{1}\right)}{\partial \Delta_{1}}= & H\left(\hat{\varepsilon}_{h}\right) f\left(\hat{\varepsilon}_{h}+\Delta_{1}\right)-H\left(-\hat{\varepsilon}_{h}\right) f\left(-\hat{\varepsilon}_{h}+\Delta_{1}\right)-\hat{\varepsilon}_{h} f\left(\hat{\varepsilon}_{h}+\Delta_{1}\right) \\
& -\int_{-\hat{\varepsilon}_{h}+\Delta}^{\hat{\varepsilon}_{h}+\Delta} H^{\prime}\left(t-\Delta_{1}\right) f(t) d t-\int_{\hat{\varepsilon}_{h}+\Delta_{1}}^{\infty} f(t) d t \\
= & -\int_{-\hat{\varepsilon}_{h}+\Delta_{1}}^{\hat{\varepsilon}_{h}+\Delta_{1}} F\left(t-\Delta_{1}\right) f(t) d t-\left[1-F\left(\hat{\varepsilon}_{h}+\Delta_{1}\right)\right]<0
\end{aligned}
$$

where the second equality follows from the facts that $H\left(\hat{\varepsilon}_{h}\right)=\hat{\varepsilon}_{h}, H\left(-\hat{\varepsilon}_{h}\right)=0$ and $H^{\prime}=F$.

Note that

$$
\begin{equation*}
\lim _{\Delta_{1} \rightarrow \infty} B\left(\Delta_{1}\right)=0 \tag{22}
\end{equation*}
$$

and

$$
\begin{equation*}
\lim _{\Delta_{1} \rightarrow 0} B\left(\Delta_{1}\right)=\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x) f(x) d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x f(x) d x \tag{23}
\end{equation*}
$$

Both $\hat{\varepsilon}_{h}$ and $H(x)$ depend on $c_{2}$. It is easy to see that $\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x) f(x) d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x f(x) d x$ decreases with $c_{2}$. When $c_{2}$ approaches the upper bound $\int_{0}^{\infty} t d F(t)$ (see Assumption 1), $\hat{\varepsilon}_{h} \rightarrow 0$ so that $\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} H(x) f(x) d x+\int_{\hat{\varepsilon}_{h}}^{\infty} x f(x) d x \rightarrow \int_{0}^{\infty} t d F(t)$. According to (22), (23) and Assumption 1, we have

$$
\lim _{\Delta_{1} \rightarrow \infty} B\left(\Delta_{1}\right)<c_{1}<\int_{0}^{\infty} t d F(t)<\lim _{\Delta_{1} \rightarrow 0} B\left(\Delta_{1}\right)
$$

Since $\frac{\partial B\left(\Delta_{1}\right)}{\partial \Delta_{1}}<0$ for $\Delta_{1} \geq 0$, there exists a unique $\Delta_{0}>0$ such that $B\left(\Delta_{0}\right)=c_{1}$, which satisfies that $B\left(\Delta_{1}\right)<c_{1}$ if and only if $\Delta_{1}>\Delta_{0}$ (given that $\Delta_{1} \geq 0$ ). The symmetry of $B\left(\Delta_{1}\right)$ implies that, for all $\Delta_{1} \in(-\infty, \infty), B\left(\Delta_{1}\right)<c_{1}$ if and only if $\left|\Delta_{1}\right|>\Delta_{0}$, which completes our proof.

Proof of Lemma 2. Recall that for any $\Delta_{i} \in\left[-\Delta_{0}, \Delta_{0}\right]$ :

$$
\begin{aligned}
& q_{i}^{i}\left(\Delta_{i}\right)=\left[1-F\left(\hat{\varepsilon}_{h}-\Delta_{i}\right)\right]+\int_{-\hat{\varepsilon}_{h}-\Delta_{i}}^{\hat{\varepsilon}_{h}-\Delta_{i}} F\left(x+\Delta_{i}\right) d F(x) \\
& q_{i}^{j}\left(\Delta_{i}\right)=F\left(\Delta_{i}-\hat{\varepsilon}_{h}\right)+\int_{\Delta_{i}-\hat{\varepsilon}_{h}}^{\Delta_{i}+\hat{\varepsilon}_{h}}\left[1-F\left(x-\Delta_{i}\right)\right] d F(x)
\end{aligned}
$$

Applying integration by substitution, we can rewrite $q_{i}^{i}\left(\Delta_{i}\right)$ and $q_{i}^{j}\left(\Delta_{i}\right)$ as follows:

$$
\begin{aligned}
& q_{i}^{i}\left(\Delta_{i}\right)=\left[1-F\left(\hat{\varepsilon}_{h}-\Delta_{i}\right)\right]+\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} F(t) f\left(t-\Delta_{i}\right) d t \\
& q_{i}^{j}\left(\Delta_{i}\right)=F\left(\Delta_{i}-\hat{\varepsilon}_{h}\right)+\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}}[1-F(t)] f\left(t+\Delta_{i}\right) d t
\end{aligned}
$$

Using $x=-t$ and applying integration by substitution, for any $\Delta_{i} \in\left[-\Delta_{0}, \Delta_{0}\right]$ :

$$
\begin{equation*}
\int_{-\hat{\varepsilon}_{h}}^{0} F(t) f\left(t-\Delta_{i}\right) d t=\int_{0}^{\hat{\varepsilon}_{h}} F(-x) f\left(-x-\Delta_{i}\right) d x=\int_{0}^{\hat{\varepsilon}_{h}}[1-F(x)] f\left(x+\Delta_{i}\right) d x \tag{24}
\end{equation*}
$$

and

$$
\begin{equation*}
\int_{-\hat{\varepsilon}_{h}}^{0}[1-F(t)] f\left(t+\Delta_{i}\right) d t=\int_{0}^{\hat{\varepsilon}_{h}}[1-F(-x)] f\left(-x+\Delta_{i}\right) d x=\int_{0}^{\hat{\varepsilon}_{h}} F(x) f\left(x-\Delta_{i}\right) d x \tag{25}
\end{equation*}
$$

where we have repeatedly used $F(x)=1-F(-x)$ and the symmetry of $f$.
Expressions (24) and (25) imply that:
$\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}} F(t) f\left(t-\Delta_{i}\right) d t=\int_{-\hat{\varepsilon}_{h}}^{\hat{\varepsilon}_{h}}[1-F(t)] f\left(t+\Delta_{i}\right) d t$,
which in turn implies that $q_{i}^{i}\left(\Delta_{i}\right)-q_{i}^{j}\left(\Delta_{i}\right)=0$, for any $\Delta_{i} \in\left[-\Delta_{0}, \Delta_{0}\right]$.
Proof of Proposition 3. We prove this proposition in three steps: (i) we show that the demand function $D_{i}(\mathbf{p})$ is logconcave in $p_{i}$, (ii) we demonstrate the existence of the equilibrium and (iii) we demonstrate the uniqueness of the equilibrium.

In step (i), since the log-concavity is preserved by the independent summation of log-concave distributed random variables, the fact that $g$ is log-concave implies that both $k$ and $K$ are log-concave. We next show that the demand function

$$
S_{i}\left(\Delta_{i}\right)=\left\{\begin{array}{cc}
0 & \text { if } \Delta_{i}<-\Delta_{0} \\
{\left[1-F\left(\hat{\varepsilon}_{h}-\Delta_{i}\right)\right]+\int_{-\hat{\varepsilon}_{h}-\Delta_{i}}^{\hat{\varepsilon}_{h}-\Delta_{i}} F\left(x+\Delta_{i}\right) d F(x)} & \text { if }-\Delta_{0} \leq \Delta_{i} \leq \Delta_{0} \\
1 & \text { if } \Delta_{i}>\Delta_{0}
\end{array}\right.
$$

is log-concave in $\Delta_{i}$.
The expression of $S_{i}\left(\Delta_{i}\right)$ can be rewritten as:

$$
S_{i}\left(\Delta_{i}\right)=\left\{\begin{array}{cc}
0 & \text { if } \Delta_{i}<-\Delta_{0}  \tag{26}\\
\int_{-\infty}^{\infty} \mu\left(t+\Delta_{i}\right) f(t) d t & \text { if }-\Delta_{0} \leq \Delta_{i} \leq \Delta_{0} \\
1 & \text { if } \Delta_{i}>\Delta_{0}
\end{array}\right.
$$

where

$$
\mu(y)=\left\{\begin{array}{cc}
0 & \text { if } y<-\hat{\varepsilon}_{h} \\
F(y) & \text { if }-\hat{\varepsilon}_{h} \leq y \leq \hat{\varepsilon}_{h} \\
1 & \text { if } y>\hat{\varepsilon}_{h}
\end{array}\right.
$$

Since both $\mu$ and $f$ are log-concave, $S_{i}\left(\Delta_{i}\right)$ is log-concave in $\Delta_{i}$. Finally, since both $S_{i}$ and $K$ are log-concave, the expected demand $D_{i}(\mathbf{p})$, given by (12), is log-concave in $p_{i}$.

In step (ii), first note that $p_{1}=p_{2}=p^{*}$, which is given by (15), satisfies the first-order condition (14) for each firm, where we used the fact that $D_{i}(\delta=0)=\frac{1}{2}$. We prove that both firms charging price $p^{*}$ constitute a pricing equilibrium. That is, given $p_{j}=p^{*}$, firm $i$ 's profit $\pi\left(p_{i}\right)=p_{i} D_{i}\left(p_{i}, p^{*}\right)$ is maximized when $p_{i}=p^{*}$.

Note that

$$
\begin{aligned}
\pi^{\prime}\left(p_{i}\right) & =D_{i}\left(p_{i}, p^{*}\right)+p_{i} \frac{\partial D_{i}\left(p_{i}, p^{*}\right)}{\partial p_{i}} \\
& =\frac{\partial D_{i}\left(p_{i}, p^{*}\right)}{\partial p_{i}} \theta\left(p_{i}\right)
\end{aligned}
$$

where

$$
\theta\left(p_{i}\right) \equiv\left[p_{i}+\frac{D_{i}\left(p_{i}, p^{*}\right)}{\frac{\partial D_{i}\left(p_{i}, p^{*}\right)}{\partial p_{i}}}\right]
$$

which is increasing in $p_{i}$ due to the log-concavity of $D_{i}\left(p_{i}, p_{j}^{*}\right)$ in $p_{i}$. Moreover, since $\theta\left(p^{*}\right)=0$, it must be that $\theta\left(p_{i}\right)<0$ when $p_{i}<p^{*}$ and $\theta\left(p_{i}\right)>0$ when $p_{i}>p^{*}$. Since $\frac{\partial D_{i}\left(p_{i}, p^{*}\right)}{\partial p_{i}}<0, \pi^{\prime}\left(p_{i}\right)>0$ when $p_{i}<p^{*}$ and $\pi^{\prime}\left(p_{i}\right)<0$ when $p_{i}>p^{*}$. Therefore, $p^{*}$ indeed maximizes $\pi\left(p_{i}\right)=p_{i} D_{i}\left(p_{i}, p^{*}\right)$.

In step (iii), the uniqueness of the equilibrium follows from the fact that each firm's demand only depends on the price difference. Specifically, let ( $\hat{p}_{1}, \hat{p}_{2}$ ) be any price equilibrium. If $\hat{p}_{1}=\hat{p}_{2}$, according to the first-order condition (14), it must be that $\hat{p}_{1}=\hat{p}_{2}=p^{*}$, which is given by (15).

If $\hat{p}_{1} \neq \hat{p}_{2}$, without loss of generality, assume $\delta=\hat{p}_{1}-\hat{p}_{2}>0$. The first-order condition gives

$$
\hat{p}_{1}=-\frac{D_{1}(\delta)}{D_{1}^{\prime}(\delta)}
$$

Note that due to the symmetric setting, $\left(\hat{p}_{2}, \hat{p}_{1}\right)$ also constitutes a price equilibrium, which implies that

$$
\hat{p}_{2}=-\frac{D_{1}(-\delta)}{D_{1}^{\prime}(-\delta)}
$$

which is impossible because $\hat{p}_{1}>\hat{p}_{2}$ and $-\frac{D_{1}(\delta)}{D_{1}^{\prime}(\delta)}<-\frac{D_{1}(-\delta)}{D_{1}^{\prime}(-\delta)}$ due to log-concavity of $D_{1}(\delta)$. This implies that an asymmetric equilibrium does not exist. As such, the only symmetric equilibrium is given by (15). That is

$$
\begin{aligned}
p^{*} & =-\frac{\left.D_{i}\left(\delta_{i}\right)\right|_{\delta_{i}=0}}{\left.D_{i}^{\prime}\left(\delta_{i}\right)\right|_{\delta_{i}=0}}=-\frac{1}{2\left[\int_{-\Delta_{0}}^{\Delta_{0}} S_{i}(x) k^{\prime}(x) d x-k\left(\Delta_{0}\right)\right]} \\
& =-\frac{1}{2\left[\int_{-\Delta_{0}}^{0} S_{i}(x) k^{\prime}(x) d x+\int_{0}^{\Delta_{0}} S_{i}(x) k^{\prime}(x) d x-k\left(\Delta_{0}\right)\right]} \\
& =-\frac{1}{2\left[\int_{0}^{\Delta_{0}} S_{i}(-x) k^{\prime}(-x) d x+\int_{0}^{\Delta_{0}} S_{i}(x) k^{\prime}(x) d x-k\left(\Delta_{0}\right)\right]} \\
& =\frac{1}{-2 k(0)+4\left[k\left(\Delta_{0}\right)-\int_{0}^{\Delta_{0}} S_{i}(x) k^{\prime}(x) d x\right]}
\end{aligned}
$$

where we applied integration by substitution and used the fact that $S_{i}(x)+S_{i}(-x)=1$ for all $x$.
The following result is useful in proving Propositions 4-6.
Result: Suppose $f$ is a continuously differentiable density function. If $f$ is log-concave, then $f$ is either monotone or single-peaked.

Proof. Suppose $f$ is not monotone. Since $f$ is continuously differentiable, there must exist $x_{0}$ such that $f^{\prime}\left(x_{0}\right)=0$. Due to log-concavity, $\frac{f^{\prime}(x)}{f(x)}$ is nondecreasing in $x$, which means

$$
\frac{f^{\prime}(x)}{f(x)} \leq \frac{f^{\prime}\left(x_{0}\right)}{f\left(x_{0}\right)}=0, \text { for any } x>x_{0}
$$

and

$$
\frac{f^{\prime}(x)}{f(x)} \geq \frac{f^{\prime}\left(x_{0}\right)}{f\left(x_{0}\right)}=0, \text { for any } x<x_{0}
$$

Since $f(x)>0$, this implies that $f$ is single-peaked, with $f^{\prime}(x) \geq 0$ for $x<x_{0}$, and $f^{\prime}(x) \leq 0$ for $x>x_{0}$.
According to the above result, if $f$ is both symmetric and log-concave, $f$ must be single-peaked, with $f^{\prime}(x) \geq 0$ for $x<0$, and $f^{\prime}(x) \leq 0$ for $x>0$ (because symmetry implies that $f^{\prime}(0)=0$ ).

Proof of Proposition 4. Recall that the equilibrium prices for the two models are

$$
\begin{aligned}
p^{*} & =\frac{1}{-2 k(0)+4\left[k\left(\Delta_{0}\right)-\int_{0}^{\Delta_{0}} S_{i}(x) k^{\prime}(x) d x\right]} \\
p_{n}^{*} & =-\frac{1}{2 k(0)+4 \int_{0}^{\infty} S_{i}^{n}(x) k^{\prime}(x) d x}
\end{aligned}
$$

We have that

$$
\begin{equation*}
\frac{1}{p^{*}}-\frac{1}{p_{n}^{*}}=-4 \int_{0}^{\Delta_{0}}\left[S_{i}(x)-S_{i}^{n}(x)\right] k^{\prime}(x) d x-4 \int_{\Delta_{0}}^{\infty}\left[1-S_{i}^{n}(x)\right] k^{\prime}(x) d x \tag{27}
\end{equation*}
$$

Given $c_{2}$ (and $\hat{\varepsilon}_{h}$ ), the difference $\frac{1}{p^{*}}-\frac{1}{p_{n}^{*}}$ depends only on $\Delta_{0}$ (or $c_{1}$ ). From (27), we have

$$
\frac{\partial\left[\frac{1}{p^{*}}-\frac{1}{p_{n}^{*}}\right]}{\partial \Delta_{0}}=4 k^{\prime}\left(\Delta_{0}\right)\left[1-S_{i}\left(\Delta_{0}\right)\right]<0
$$

Note that for $x \in\left[0, \Delta_{0}\right]$,

$$
\begin{aligned}
& S_{i}(x)=\left[1-F\left(\hat{\varepsilon}_{h}-x\right)\right]+\int_{-\hat{\varepsilon}_{h}-x}^{\hat{\varepsilon}_{h}-x} F(t+x) d F(t) \\
& S_{i}^{n}(x)=\left[1-F\left(\hat{\varepsilon}_{h}-x\right)\right]+\int_{-\infty}^{\hat{\varepsilon}_{h}-x} F(t+x) d F(t)
\end{aligned}
$$

Thus, $S_{i}(x)-S_{i}^{n}(x)=-\int_{-\infty}^{-\hat{\varepsilon}_{h}-x} F(t+x) d F(t)<0$.
Since $k^{\prime}(x) \leq 0$ for all $x>0$, from (27), we have

$$
\lim _{\Delta_{0} \rightarrow 0} \frac{1}{p^{*}}-\frac{1}{p_{n}^{*}}=-4 \int_{0}^{\infty}\left[1-S_{i}^{n}(x)\right] k^{\prime}(x) d x>0
$$

and

$$
\lim _{\Delta_{0} \rightarrow \infty} \frac{1}{p^{*}}-\frac{1}{p_{n}^{*}}=-4 \int_{0}^{\infty}\left[S_{i}(x)-S_{i}^{n}(x)\right] k^{\prime}(x) d x<0
$$

Since $\frac{\partial \Delta_{0}}{\partial c_{1}}<0$, there exists $\hat{c}_{1}$ such that, $\frac{1}{p^{*}}<\frac{1}{p_{n}^{*}}$ (or $p^{*}>p_{n}^{*}$ ) if and only if $c_{1}<\hat{c}_{1}$.
Proof of Proposition 5. The equilibrium price under blind buying is

$$
\begin{equation*}
p^{*}=\frac{1}{-2 k(0)+4\left[k\left(\Delta_{0}\right)-\int_{0}^{\Delta_{0}} S_{i}(x) k^{\prime}(x) d x\right]} \tag{28}
\end{equation*}
$$

Consider $p^{*}$ as a function of $\left(\Delta_{0}, \hat{\varepsilon}_{h}\right)$ (note that $S_{i}(x)$ depends on $\left.\hat{\varepsilon}_{h}\right)$, in spite of the fact that $\Delta_{0}$ also depends on $\hat{\varepsilon}_{h}$. We first study the effect of $c_{1}$ on the equilibrium price $p^{*}$. Note that $c_{1}$ affects $p^{*}$ only through $\Delta_{0}$. We have that

$$
\frac{\partial\left[k\left(\Delta_{0}\right)-\int_{0}^{\Delta_{0}} S_{i}(x) k^{\prime}(x) d x\right]}{\partial \Delta_{0}}=k^{\prime}\left(\Delta_{0}\right)\left[1-S_{i}\left(\Delta_{0}\right)\right] \leq 0
$$

where we have used the fact that $k$ is single-peaked (since $k$ is symmetric and log-concave), so that $k^{\prime}(x) \leq 0$ for all $x>0$. According to (28), this implies that $\frac{\partial p^{*}}{\partial \Delta_{0}} \geq 0$. Finally, since $\Delta_{0}$ decreases with $c_{1}$ according to the proof of Proposition 2 , we have that $\frac{\partial p^{*}}{\partial c_{1}} \leq 0$, that is, the market price decreases with the first-stage search cost.

Now we study the effect of $c_{2}$ on the equilibrium price. Note that $c_{2}$ affects $p^{*}$ through both $\Delta_{0}$ and $\hat{\varepsilon}_{h}$. We have

$$
\begin{equation*}
\frac{\partial p^{*}}{\partial c_{2}}=\frac{\partial p^{*}}{\partial \Delta_{0}} \frac{\partial \Delta_{0}}{\partial c_{2}}+\frac{\partial p^{*}}{\partial \hat{\varepsilon}_{h}} \frac{\partial \hat{\varepsilon}_{h}}{\partial c_{2}} \tag{29}
\end{equation*}
$$

From the proof of Proposition 2, we have $\frac{\partial \Delta_{0}}{\partial c_{2}}<0$. And we have $\frac{\partial \hat{\varepsilon}_{h}}{\partial c_{2}}<0$ according to the definition of $\hat{\varepsilon}_{h}$.
From (11) we have that $\frac{\partial S_{i}(x)}{\partial \hat{\varepsilon}_{h}}=F\left(-\hat{\varepsilon}_{h}\right)\left[f\left(\hat{\varepsilon}_{h}+x\right)-f\left(\hat{\varepsilon}_{h}-x\right)\right]$, which is negative for any $x>0$ since $f$ is singlepeaked, that is, $f^{\prime}(x) \geq 0$ for $x<0$, and $f^{\prime}(x) \leq 0$ for $x>0$. Since $k^{\prime}(x) \leq 0$ for all $x>0$, we have that $\frac{\partial p^{*}}{\partial \hat{\varepsilon}_{h}} \geq 0$. Finally, since $\frac{\partial p^{*}}{\partial \Delta_{0}} \geq 0$, from (29) we have that $\frac{\partial p^{*}}{\partial c_{2}} \leq 0$, i.e., the market price decreases with the second-stage search cost.

Proof of Proposition 6. The comparison of industry profits follows immediately from Proposition 4.
Let $E U^{b}\left(p_{1}, p_{2}\right)$ and $E U^{n}\left(p_{1}, p_{2}\right)$ denote the consumer's expected utility under price vector $\left(p_{1}, p_{2}\right)$ for cases with and without blind buying, respectively. Note that at a symmetric equilibrium, consumers behave exactly the same as if the prices are $p_{1}=p_{2}=0$. Thus, the total welfares for the two models are

$$
\begin{aligned}
& T W^{b}=E U^{b}(0,0) \\
& T W^{n}=E U^{n}(0,0)
\end{aligned}
$$

We first solve $E U^{b}(0,0)$. When blind buying is available, since the consumer is always indifferent of the search order, we assume the sampling order is the same as that when blind buying is not allowed. That is, (i) the consumer first samples product $i$ if $0 \leq \eta_{i}-\eta_{j} \leq \Delta_{0}$, and (ii) a blindly buying takes place when $\left|\eta_{i}-\eta_{j}\right|>\Delta_{0}$, for any $i=1,2$ and $j \in\{1,2\} \backslash\{i\}$.

Using the optimal search strategy, we can solve the consumer's expected continuation payoff given ( $\eta_{1}, \eta_{2}$ ), as follows:

$$
u^{b}\left(\eta_{1}, \eta_{2}\right)=\left\{\begin{array}{cc}
\eta_{1} & \text { if } \eta_{1}-\eta_{2}>\Delta_{0}  \tag{30}\\
-c_{1}+\eta_{2}+E_{\tilde{\varepsilon}} \max \{\gamma+\tilde{\varepsilon}, 0, H(\gamma+\tilde{\varepsilon})\} & \text { if } 0 \leq \eta_{1}-\eta_{2} \leq \Delta_{0} \\
-c_{1}+\eta_{1}+E_{\tilde{\varepsilon}} \max \{-\gamma+\tilde{\varepsilon}, 0, H(-\gamma+\tilde{\varepsilon})\} & \text { if }-\Delta_{0} \leq \eta_{1}-\eta_{2} \leq 0 \\
\eta_{2} & \text { if } \eta_{1}-\eta_{2}<-\Delta_{0}
\end{array}\right.
$$

where $\gamma=\eta_{1}-\eta_{2}, \tilde{\varepsilon}$ is the first-sampled product's match value, and

$$
H(x) \equiv-c_{2}+E_{\tilde{\varepsilon}} \max (x, \tilde{\varepsilon})=-c_{2}+x+\int_{x}^{\infty}(t-x) d F(t)
$$

Therefore, the consumer's ex-ante expected payoff with blind buying is $E U^{b}(0,0)=E_{\eta_{1}, \eta_{2}} u^{b}\left(\eta_{1}, \eta_{2}\right)$.
Similarly, when blind buying is not allowed, the consumer's expected continuation payoff given $\left(\eta_{1}, \eta_{2}\right)$ is

$$
u^{n}\left(\eta_{1}, \eta_{2}\right)=\left\{\begin{array}{cl}
-c_{1}+\eta_{2}+E_{\tilde{\varepsilon}} \max \{\gamma+\tilde{\varepsilon}, H(\gamma+\tilde{\varepsilon})\} & \text { if } \eta_{1} \geq \eta_{2}  \tag{31}\\
-c_{1}+\eta_{1}+E_{\tilde{\varepsilon}} \max \{-\gamma+\tilde{\varepsilon}, H(-\gamma+\tilde{\varepsilon})\} & \text { if } \eta_{1}<\eta_{2}
\end{array}\right.
$$

Thus, the consumer's ex-ante expected payoff without blind buying is $E U^{n}(0,0)=E_{\eta_{1}, \eta_{2}} u^{n}\left(\eta_{1}, \eta_{2}\right)$.
Define $\Delta T W(\gamma) \equiv u^{b}\left(\eta_{1}, \eta_{2}\right)-u^{n}\left(\eta_{1}, \eta_{2}\right)$, which depends only on $\gamma=\eta_{1}-\eta_{2}$. From (30) and (31), we have

$$
\Delta T W(\gamma)=\left\{\begin{array}{cc}
c_{1}+\gamma-E_{\tilde{\varepsilon}} \max \{\gamma+\tilde{\varepsilon}, H(\gamma+\tilde{\varepsilon})\} & \text { if } \gamma>\Delta_{0} \\
E_{\tilde{\varepsilon}} \max \{\gamma+\tilde{\varepsilon}, 0, H(\gamma+\tilde{\varepsilon})\}-E_{\tilde{\varepsilon}} \max \{\gamma+\tilde{\varepsilon}, H(\gamma+\tilde{\varepsilon})\} & \text { if } 0 \leq \gamma \leq \Delta_{0} \\
E_{\tilde{\varepsilon}} \max \{-\gamma+\tilde{\varepsilon}, 0, H(-\gamma+\tilde{\varepsilon})\}-E_{\tilde{\varepsilon}} \max \{-\gamma+\tilde{\varepsilon}, H(-\gamma+\tilde{\varepsilon})\} & \text { if }-\Delta_{0} \leq \gamma \leq 0 \\
c_{1}-\gamma-E_{\tilde{\varepsilon}} \max \{-\gamma+\tilde{\varepsilon}, H(-\gamma+\tilde{\varepsilon})\} & \text { if } \gamma<-\Delta_{0}
\end{array} .\right.
$$

It is obvious that $\Delta T W(\gamma) \geq 0$ when $-\Delta_{0} \leq \gamma \leq \Delta_{0}$. When $\gamma>\Delta_{0}$, we have

$$
\begin{aligned}
\Delta T W(\gamma) & =c_{1}+\gamma-E_{\tilde{\varepsilon}} \max \{\gamma+\tilde{\varepsilon}, H(\gamma+\tilde{\varepsilon})\} \\
& \geq c_{1}+\gamma-E_{\tilde{\varepsilon}} \max \{\gamma+\tilde{\varepsilon}, 0, H(\gamma+\tilde{\varepsilon})\} \\
& >0
\end{aligned}
$$

where the last inequality follows from the definition of $\Delta_{0}$. Similarly, When $\gamma<-\Delta_{0}$,

$$
\begin{aligned}
\Delta T W(\gamma) & =c_{1}-\gamma-E_{\tilde{\varepsilon}} \max \{-\gamma+\tilde{\varepsilon}, H(-\gamma+\tilde{\varepsilon})\} \\
& \geq c_{1}-\gamma-E_{\tilde{\varepsilon}} \max \{-\gamma+\tilde{\varepsilon}, 0, H(-\gamma+\tilde{\varepsilon})\} \\
& >0
\end{aligned}
$$

Since $\Delta T W(\gamma) \geq 0$ for any $\gamma$, we have that $E U^{b}(0,0)>E U^{n}(0,0)$, which proves that $T W^{b}>T W^{n}$, i.e., allowing blind buying always increases total welfare.

We next investigate the effect of blind buying on consumer surplus. Note that

$$
\begin{aligned}
& C S^{b}=E U^{b}\left(p^{*}, p^{*}\right)=E U^{b}(0,0)-p^{*} \\
& C S^{n}=E U^{n}\left(p_{n}^{*}, p_{n}^{*}\right)=E U^{n}(0,0)-p_{n}^{*}
\end{aligned}
$$

We have

$$
\begin{align*}
C S^{b}-C S^{n} & =\left[E U^{b}(0,0)-p^{*}\right]-\left[E U^{n}(0,0)-p_{n}^{*}\right] \\
& =\int_{-\infty}^{\infty} \Delta T W(\gamma) d K(\gamma)-\left(p^{*}-p_{n}^{*}\right) \tag{32}
\end{align*}
$$

According to Proposition 4, when $c_{1}$ is large, we have $p^{*}<p_{n}^{*}$, so that $C S^{b}-C S^{n}>0$. In this case, allowing blind buying increases total welfare and decreases industry profit, thus increasing consumer surplus. On the other hand, when $c_{1}$ is small, we have $p^{*}>p_{n}^{*}$. In this case, allowing blind buying increases both total welfare and industry profit, and the effect on consumer surplus is ambiguous. We use the following example to show that consumer surplus can decrease when $c_{1}$ is small.

Example: Consider $c_{1} \rightarrow 0$ (so that $\Delta_{0} \rightarrow \infty$ ). Suppose $\gamma \equiv \eta_{1}-\eta_{2}$ is symmetrically distributed on interval [ $-a, a$ ], where $a>0$. For each $m>0$, define $\eta_{i}^{m} \equiv m \eta_{i}$ for $i=1,2$, and $\gamma^{m} \equiv \eta_{1}^{m}-\eta_{2}^{m}$. Since $\gamma^{m}=m \gamma, \gamma^{m}$ is symmetrically distributed on interval [-ma, ma], with density $k^{m}(x)=\frac{k(x / m)}{m}$ and distribution function $K^{m}(x)=K(x / m)$. We reconsider the market game by replacing $\eta_{i}^{m}$ with $\eta_{i}$ for $i=1,2$, and $\gamma^{m}$ with $\gamma$.

Then, for each $m>0$, the equilibrium price with blind buying is given as follows (note that $\Delta_{0} \rightarrow \infty$ )

$$
\begin{aligned}
p^{*}(m) & =\frac{1}{4 \int_{0}^{m a} S_{i}^{\prime}(x) k^{m}(x) d x} \\
& =\frac{1}{4 \int_{0}^{m a} S_{i}^{\prime}(x) \frac{k(x / m)}{m} d x} \\
& =\frac{1}{4 \int_{0}^{a} S_{i}^{\prime}(m x) k(x) d x}
\end{aligned}
$$

The equilibrium price without blind buying is given as follows.

$$
\begin{aligned}
p_{n}^{*}(m) & =\frac{1}{2 k^{m}(0)\left[1-F\left(\hat{\varepsilon}_{h}\right)\right]^{2}+4 \int_{0}^{m a} S_{i}^{n \prime}(x) k^{m}(x) d x} \\
& =\frac{1}{2 \frac{k(0)}{m}\left[1-F\left(\hat{\varepsilon}_{h}\right)\right]^{2}+4 \int_{0}^{a} S_{i}^{\prime \prime}(m x) k(x) d x}
\end{aligned}
$$

Finally, the welfare difference is

$$
\begin{aligned}
\int_{-m a}^{m a} \Delta T W(x) d K^{m}(x) & =2 \int_{0}^{m a} \Delta T W(x) \frac{k(x / m)}{m} d x \\
& =2 \int_{0}^{a} \Delta T W(m x) k(x) d x
\end{aligned}
$$

Note that, when $m \rightarrow 0$, the above expressions imply that

$$
\begin{aligned}
& \lim _{m \rightarrow 0} p^{*}(m)=\frac{1}{2 S_{i}^{\prime}(0)} \\
& \lim _{m \rightarrow 0} p_{n}^{*}(m)=0 \\
& \lim _{m \rightarrow 0} \int_{-m a}^{m a} \Delta T W(x) d K^{m}(x)=\Delta T W(0)
\end{aligned}
$$

Thus, when $m \rightarrow 0$, the difference in the consumer surplus (32) is:

$$
\lim _{m \rightarrow 0} C S^{b}-C S^{n}=\Delta T W(0)-\frac{1}{2 S_{i}^{\prime}(0)}=\int_{-\infty}^{-\hat{\varepsilon}_{h}} F^{2}(x) d x-\frac{1}{4\left[F\left(-\hat{\varepsilon}_{h}\right) f\left(\hat{\varepsilon}_{h}\right)+\int_{0}^{\hat{\varepsilon}_{h}} f^{2}(x) d x\right]}
$$

so that

$$
\begin{equation*}
\lim _{\hat{\varepsilon}_{h} \rightarrow 0} \lim _{m \rightarrow 0} C S^{b}-C S^{n}=\int_{-\infty}^{0} F^{2}(x) d x-\frac{1}{2 f(0)} \tag{33}
\end{equation*}
$$

Suppose distribution $F$ satisfies $\int_{-\infty}^{0} F^{2}(x) d x-\frac{1}{2 f(0)}<0$ (this is true when $F$ is a uniform distribution). Then, according (33), $C S^{b}-C S^{n}$ is negative when $c_{1}, m$ and $\hat{\varepsilon}_{h}$ are small.

Proof of Lemma 5. Because $D_{1}(\mathbf{p})=1-D_{2}(\mathbf{p})$, it suffices to prove that $D_{2}(\mathbf{p}) \geq 1 / 2$ iff $\alpha<1 / 2$. By setting $\delta_{1}=0$, (19) becomes:

$$
\begin{align*}
D_{2}(\mathbf{p}) & =\alpha\left[(1-\alpha) K\left(\frac{c_{2}}{1-\alpha}-r\right)+\alpha K\left(-\frac{c_{2}}{\alpha}\right)\right]  \tag{34}\\
& +(1-\alpha)\left[(1-\alpha) K\left(\frac{c_{2}}{1-\alpha}\right)+\alpha K\left(r-\frac{c_{2}}{\alpha}\right)\right]
\end{align*}
$$

Recall that $\gamma_{1}=\eta_{1}-\eta_{2}$, where $\eta_{1}$ and $\eta_{2}$ are i.i.d. Therefore, $\gamma$ is symmetrically distributed and $K(0)=1 / 2, K(\gamma)=$ $1-K(-\gamma), k(\gamma)=k(-\gamma)$. Then, it is easy to derive that $\left.D_{2}(\mathbf{p})\right|_{\alpha=1 / 2}=1 / 2$.

Define $l\left(c_{2}, \alpha\right) \equiv \alpha K\left(-\frac{c_{2}}{\alpha}\right)+(1-\alpha) K\left(\frac{c_{2}}{1-\alpha}\right)$. We prove that $l\left(c_{2}, \alpha\right) \geq 1 / 2$ iff $\alpha \leq 1 / 2$, which is useful for the subsequent proof. We have:

$$
\frac{\partial l}{\partial c_{2}}=k\left(\frac{c_{2}}{1-\alpha}\right)-k\left(\frac{c_{2}}{\alpha}\right) .
$$

Since $k(x)$ is decreasing when $x \geq 0, \frac{\partial l}{\partial c_{2}} \geq 0$ iff $\alpha \leq 1 / 2$. Note that $l\left(c_{2}=0, \alpha\right)=1 / 2$ for any $\alpha$. Therefore, $l\left(c_{2}, \alpha\right) \geq 1 / 2$ iff $\alpha \leq 1 / 2$.

Assume $\alpha \leq 1 / 2$. Due to Assumption 2, we have

$$
\frac{c_{2}}{1-\alpha}<\max \left\{\frac{c_{2}}{\alpha}, r-\frac{c_{2}}{\alpha}\right\}<r-\frac{c_{2}}{1-\alpha}
$$

Notice that $K$ is concave in $\gamma$ for all $\gamma>0$, which implies that

$$
K\left(\frac{c_{2}}{\alpha}\right)-K\left(\frac{c_{2}}{1-\alpha}\right) \geq K\left(r-\frac{c_{2}}{1-\alpha}\right)-K\left(r-\frac{c_{2}}{\alpha}\right)
$$

Using the fact that $K(\gamma)=1-K(-\gamma)$, we can rewrite the above inequality as

$$
\begin{equation*}
K\left(\frac{c_{2}}{1-\alpha}-r\right)+K\left(r-\frac{c_{2}}{\alpha}\right) \geq K\left(-\frac{c_{2}}{\alpha}\right)+K\left(\frac{c_{2}}{1-\alpha}\right) \tag{35}
\end{equation*}
$$

Using (34) and (35), we have

$$
\begin{aligned}
D_{2}(\mathbf{p}) & =\alpha^{2} K\left(-\frac{c_{2}}{\alpha}\right)+(1-\alpha)^{2} K\left(\frac{c_{2}}{1-\alpha}\right) \\
& +\alpha(1-\alpha)\left[K\left(\frac{c_{2}}{1-\alpha}-r\right)+K\left(r-\frac{c_{2}}{\alpha}\right)\right] \\
& \geq \alpha^{2} K\left(-\frac{c_{2}}{\alpha}\right)+(1-\alpha)^{2} K\left(\frac{c_{2}}{1-\alpha}\right) \\
& +\alpha(1-\alpha)\left[K\left(-\frac{c_{2}}{\alpha}\right)+K\left(\frac{c_{2}}{1-\alpha}\right)\right] \\
& =\alpha K\left(-\frac{c_{2}}{\alpha}\right)+(1-\alpha) K\left(\frac{c_{2}}{1-\alpha}\right) \\
& =l\left(c_{2}, \alpha\right) \geq 1 / 2, \text { since } \alpha \leq 1 / 2
\end{aligned}
$$

Similarly one can prove that $D_{2}(\mathbf{p}) \leq 1 / 2$ when $\alpha>1 / 2$.

Proof of Proposition 7. Recall that $D_{1}(\mathbf{p})=1-D_{2}(\mathbf{p})$ and that demands only depend on the price difference $\delta_{1}=p_{1}-p_{2}$, from which we obtain:

$$
\frac{\partial D_{1}(\mathbf{p})}{\partial p_{1}}=\frac{\partial D_{2}(\mathbf{p})}{\partial p_{2}}=-\frac{\partial D_{1}(\mathbf{p})}{\partial p_{2}}=-\frac{\partial D_{2}(\mathbf{p})}{\partial p_{1}}
$$

Assume the price pair $\mathbf{p}^{*}=\left(p_{1}^{*}, p_{2}^{*}\right)$ constitutes an equilibrium. The first-order condition for $p_{i}^{*}$ of firm $i$ 's profit maximization problem implies that $D_{i}\left(\mathbf{p}^{*}\right)+p_{i}^{*} \frac{\partial D_{i}\left(\mathbf{p}^{*}\right)}{\partial p_{i}}=0$, from which we have:

$$
\frac{D_{i}\left(\mathbf{p}^{*}\right)}{p_{i}^{*}}=-\frac{\partial D_{i}\left(\mathbf{p}^{*}\right)}{\partial p_{i}}
$$

Because $\frac{\partial D_{1}(\mathbf{p})}{\partial p_{1}}=\frac{\partial D_{2}(\mathbf{p})}{\partial p_{2}}$, the above equality implies $\frac{D_{1}\left(\mathbf{p}^{*}\right)}{p_{1}^{*}}=\frac{D_{2}\left(\mathbf{p}^{*}\right)}{p_{2}^{*}}$ and, therefore:

$$
\frac{D_{1}\left(\mathbf{p}^{*}\right)}{D_{2}\left(\mathbf{p}^{*}\right)}=\frac{p_{1}^{*}}{p_{2}^{*}}
$$

The profit is $\pi_{i}^{*}=p_{i}^{*} D_{i}^{*}$. Using the fact that $\frac{D_{1}\left(\mathbf{p}^{*}\right)}{p_{1}^{*}}=\frac{D_{2}\left(\mathbf{p}^{*}\right)}{p_{2}^{*}}$, we have

$$
\frac{\pi_{1}^{*}}{\pi_{2}^{*}}=\left(\frac{p_{1}^{*}}{p_{2}^{*}}\right)^{2}
$$

Therefore, $\pi_{1}^{*} \leq \pi_{2}^{*}$ if and only if $p_{1}^{*} \leq p_{2}^{*}$.
Assume $\alpha \leq 1 / 2$. We already have that $\left.D_{2}(\mathbf{p})\right|_{\delta_{1}=0} \geq 1 / 2$, which implies $\left.D_{1}(\mathbf{p})\right|_{\Delta=0}=1-\left.D_{2}(\mathbf{p})\right|_{\delta_{1}=0} \leq 1 / 2$. Now we prove that, in equilibrium, it must be that $p_{1}^{*} \leq p_{2}^{*}$. We prove our case by contradiction. Suppose $p_{1}^{*}>p_{2}^{*}$. Then, $\delta_{1}=$ $p_{1}^{*}-p_{2}^{*}>0$. Because $D_{2}(\mathbf{p})$ is increasing in $\delta_{1}$, we have $D_{2}\left(\mathbf{p}^{*}\right) \geq\left. D_{2}(\mathbf{p})\right|_{\delta_{1}=0} \geq 1 / 2$, and $D_{1}\left(\mathbf{p}^{*}\right)=1-D_{2}\left(\mathbf{p}^{*}\right) \leq 1 / 2$. However, then $\frac{p_{1}^{*}}{p_{2}^{*}}=\frac{D_{1}\left(\mathbf{p}^{*}\right)}{D_{2}\left(\mathbf{p}^{*}\right)} \leq 1$, which is a contradiction.

Similarly we can prove that $p_{1}^{*} \geq p_{2}^{*}$ and $\pi_{1}^{*} \geq \pi_{2}^{*}$ if $\alpha>1 / 2$.

Proof of Lemma 6. Consider the case of $v_{1}<v_{2}$ (the case of $v_{2}>v_{1}$ is similar).
If a consumer decides to blind buy without any search, she will buy product 2 and obtain expected utility $v_{2}+\alpha r$.
Now suppose the consumer decides to search in stage 1 . We shall prove that $u_{1}\left(v_{1}, v_{2}\right) \leq u_{2}\left(v_{1}, v_{2}\right)$ such that the consumer will optimally choose to search firm 1 (resp. firm 2) first if $\alpha \leq 1 / 2$ (resp. $\alpha \geq 1 / 2$ ).

Suppose a consumer searches firm 1 first. Then, with probability $1-\alpha, \varepsilon_{1}=0$, and the consumer will buy product 2 blindly, and obtain a utility $v_{2}+\alpha r$.

With probability $\alpha, \varepsilon_{1}=r$. Then, we have three cases.
a) $v_{1}-v_{2}>-\frac{c_{2}}{\alpha}$. In this case, the consumer will buy product 1 immediately, and obtain interim utility $v_{1}+r$;
b) $v_{1}-v_{2}<\frac{c_{2}^{\alpha}}{1-\alpha}-r$. Then the consumer will buy product 2 blindly, and obtain interim utility $v_{2}+\alpha r$;
c) If $\frac{c_{2}}{1-\alpha}-r<v_{1}-v_{2}<-\frac{c_{2}}{\alpha}$. Then the consumer will inspect 2 , and obtain interim utility $\alpha\left(v_{2}+r\right)+(1-\alpha)\left(v_{1}+r\right)-$ $c_{2}$.

We can then derive the ex ante utility (net of search cost $c_{1}$ ) and purchasing probability in the three cases:
a) $v_{1}-v_{2}>-\frac{c_{2}}{\alpha}$. Then, $u_{1}\left(v_{1}, v_{2}\right)=\alpha\left(v_{1}+r\right)+(1-\alpha)\left(v_{2}+\alpha r\right)$. The probability of purchasing product 1 is $\alpha$.
b) $v_{1}-v_{2}<\frac{c_{2}^{\alpha}}{1-\alpha}-r$. Then, $u_{1}\left(v_{1}, v_{2}\right)=v_{2}+\alpha r$. The probability of buying product 1 is 0 .
c) $\frac{c_{2}}{1-\alpha}-r<v_{1}-v_{2}<-\frac{c_{2}}{\alpha}$. Then $u_{1}\left(v_{1}, v_{2}\right)=\alpha\left(\alpha\left(v_{2}+r\right)+(1-\alpha)\left(v_{1}+r\right)-c_{2}\right)+(1-\alpha)\left(v_{2}+\alpha r\right)$. The probability of buying product 1 is $\alpha(1-\alpha)$.

Similarly, after tedious calculation, we can derive the ex ante utility and purchasing probability when the consumer search product 2 first as follows.
a) $v_{1}-v_{2}<\frac{c_{2}}{\alpha}-r$. Then $u_{2}\left(v_{1}, v_{2}\right)=v_{2}+\alpha r$. The probability of buying product 1 is 0 .
b) $v_{1}-v_{2}>-\frac{c_{2}}{1-\alpha}$. Then $u_{2}\left(v_{1}, v_{2}\right)=\alpha\left(v_{2}+r\right)+(1-\alpha)\left(v_{1}+\alpha r\right)$. The probability of buying product 1 is $1-\alpha$.
c) $\frac{c_{2}}{\alpha}-r<v_{1}-v_{2}<-\frac{c_{2}}{1-\alpha}$. Then $u_{2}\left(v_{1}, v_{2}\right)=\alpha\left(v_{2}+r\right)+(1-\alpha)\left(\alpha\left(v_{1}+r\right)+(1-\alpha) v_{2}-c_{2}\right)$. The probability of buying product 1 is $(1-\alpha) \alpha$.

Now we prove that $u_{1}\left(v_{1}, v_{2}\right)-u_{2}\left(v_{1}, v_{2}\right) \geq 0$ if $\alpha \leq 1 / 2$. In this case, we have the order $\frac{c_{2}}{1-\alpha}-r<\frac{c_{2}}{\alpha}-r<-\frac{c_{2}}{\alpha}<$ $-\frac{c_{2}}{1-\alpha}$.

If $v_{1}-v_{2}<\frac{c_{2}}{1-\alpha}-r$, then

$$
u_{1}\left(v_{1}, v_{2}\right)-u_{2}\left(v_{1}, v_{2}\right)=0
$$

If $\frac{c_{2}}{1-\alpha}-r<v_{1}-v_{2}<\frac{c_{2}}{\alpha}-r$, then

$$
\begin{aligned}
u_{1}\left(v_{1}, v_{2}\right)-u_{2}\left(v_{1}, v_{2}\right) & =\alpha\left(\alpha\left(v_{2}+r\right)+(1-\alpha)\left(v_{1}+r\right)-c_{2}\right)+(1-\alpha)\left(v_{2}+\alpha r\right)-\left(v_{2}+\alpha r\right) \\
& =(1-\alpha)\left(v_{1}-v_{2}+r\right)-c_{2}>0
\end{aligned}
$$

If $\frac{c_{2}}{\alpha}-r<v_{1}-v_{2}<-\frac{c_{2}}{1-\alpha}$, then

$$
\begin{aligned}
u_{1}\left(v_{1}, v_{2}\right)-u_{2}\left(v_{1}, v_{2}\right) & =\alpha\left(\alpha\left(v_{2}+r\right)+(1-\alpha)\left(v_{1}+r\right)-c_{2}\right)+(1-\alpha)\left(v_{2}+\alpha r\right) \\
& -\left[\alpha\left(v_{2}+r\right)+(1-\alpha)\left(\alpha\left(v_{1}+r\right)+(1-\alpha) v_{2}-c_{2}\right)\right] \\
& =(1-2 \alpha) c_{2}>0
\end{aligned}
$$

If $-\frac{c_{2}}{\alpha}<v_{1}-v_{2}<-\frac{c_{2}}{1-\alpha}$, then

$$
\begin{aligned}
u_{1}\left(v_{1}, v_{2}\right)-u_{2}\left(v_{1}, v_{2}\right) & =\alpha\left(v_{1}+r\right)+(1-\alpha)\left(v_{2}+\alpha r\right)- \\
& {\left[\alpha\left(v_{2}+r\right)+(1-\alpha)\left(\alpha\left(v_{1}+r\right)+(1-\alpha) v_{2}-c_{2}\right)\right] } \\
& =\alpha^{2}\left(v_{1}-v_{2}+\frac{1-\alpha}{\alpha} \frac{c_{2}}{\alpha}\right)>0
\end{aligned}
$$

where the last inequality is due to $v_{1}-v_{2}>-\frac{c_{2}}{\alpha}>-\frac{1-\alpha}{\alpha} \frac{c_{2}}{\alpha}$ for $\alpha<1 / 2$.
If $v_{1}-v_{2}>-\frac{c_{2}}{1-\alpha}$, then

$$
\begin{aligned}
u_{1}\left(v_{1}, v_{2}\right)-u_{2}\left(v_{1}, v_{2}\right) & =\alpha\left(v_{1}+r\right)+(1-\alpha)\left(v_{2}+\alpha r\right)-\left[\alpha\left(v_{2}+r\right)+(1-\alpha)\left(v_{1}+\alpha r\right)\right] \\
& =(1-2 \alpha)\left(v_{2}-v_{1}\right)>0
\end{aligned}
$$

Thus, we have proved that $u_{1}\left(v_{1}, v_{2}\right) \geq u_{2}\left(v_{1}, v_{2}\right)$ if $\alpha \leq 1 / 2$.
Similarly, one can prove that $u_{1}\left(v_{1}, v_{2}\right) \leq u_{2}\left(v_{1}, v_{2}\right)$ if $\alpha \geq 1 / 2$.
Now we go back to stage 1 . Suppose $\alpha<1 / 2$, the consumer only needs to decide between searching firm 1 first and blindly buying product 2 by comparing $u_{1}\left(v_{1}, v_{2}\right)-c_{1}$ and $v_{2}+\alpha r$. After tedious calculation and using the assumption that $c_{1}+c_{2} \leq \alpha(1-\alpha) r$, one can derive that $u_{1}\left(v_{1}, v_{2}\right)-c_{1} \geq v_{2}+\alpha r$ iff $v_{2}-v_{1} \leq r-\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}$. The case of $\alpha \geq 1 / 2$ is similar.

Proof of Proposition 8. Consider the case of $\alpha<1 / 2$ (The proof for the case of $\alpha>1 / 2$ is similar). The demand of firm 1 is

$$
\begin{aligned}
D_{1} & =\alpha \operatorname{Pr}\left(0>v_{1}-v_{2}>-\frac{c_{2}}{\alpha}\right)+\alpha(1-\alpha) \operatorname{Pr}\left(-\left(r-\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}\right)<v_{1}-v_{2}<-\frac{c_{2}}{\alpha}\right) \\
& +(1-\alpha) \operatorname{Pr}\left(0<v_{1}-v_{2}<\frac{c_{2}}{\alpha}\right)+\operatorname{Pr}\left(v_{1}-v_{2}>r-\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}\right) \\
& +(1-\alpha(1-\alpha)) \operatorname{Pr}\left(\frac{c_{2}}{\alpha}<v_{1}-v_{2}<r-\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}\right)
\end{aligned}
$$

Recall that $v_{1}-v_{2}=\gamma_{1}-\delta_{1}$. Then,

$$
\begin{aligned}
D_{1} & =\alpha\left(K\left(\delta_{1}\right)-K\left(\delta_{1}-\frac{c_{2}}{\alpha}\right)\right)+\alpha(1-\alpha)\left(K\left(\delta_{1}-\frac{c_{2}}{\alpha}\right)-K\left(\delta_{1}+\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}-r\right)\right) \\
& +(1-\alpha)\left(K\left(\delta_{1}+\frac{c_{2}}{\alpha}\right)-K\left(\delta_{1}\right)\right)+\left(1-K\left(\delta_{1}+r-\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}\right)\right) \\
& +(1-\alpha(1-\alpha))\left(K\left(\delta_{1}+r-\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}\right)-K\left(\delta_{1}+\frac{c_{2}}{\alpha}\right)\right)
\end{aligned}
$$

It is easy to derive $\left.D_{1}\right|_{\delta_{1}=0}=\frac{1}{2}$. The first order condition of firm $i$ 's optimization problem gives

$$
D_{i}+p_{i} \frac{\partial D_{i}}{\partial p_{i}}=0
$$

In symmetric equilibrium $p_{1}=p_{2}=p^{*}$. Then, the first order condition becomes $\frac{1}{2}+p^{*} \frac{\partial D_{i}\left(\mathbf{p}^{*}\right)}{\partial p_{i}}=0$, from which we have

$$
\begin{aligned}
\frac{1}{2 p^{*}} & =-\frac{\partial D_{i}\left(\mathbf{p}^{*}\right)}{\partial p_{i}} \\
& =(1-2 \alpha)\left(k(0)-k\left(-\frac{c_{2}}{\alpha}\right)\right)+(1-2 \alpha(1-\alpha))\left(k\left(-\frac{c_{2}}{\alpha}\right)-k\left(\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}-r\right)\right)+k\left(r-\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}\right)
\end{aligned}
$$

Thus, we have

$$
\frac{\partial \frac{1}{2 p^{*}}}{\partial c_{1}}=-2 k^{\prime}\left(r-\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}\right)>0
$$

and

$$
\begin{aligned}
\frac{\partial \frac{1}{2 p^{*}}}{\partial c_{2}} & =\frac{(1-2 \alpha)}{\alpha} k^{\prime}\left(-\frac{c_{2}}{\alpha}\right)+(1-2 \alpha(1-\alpha))\left(-\frac{1}{\alpha} k^{\prime}\left(-\frac{c_{2}}{\alpha}\right)-\frac{1}{1-\alpha} k^{\prime}\left(\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}-r\right)\right) \\
& -\frac{1}{1-\alpha} k^{\prime}\left(r-\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}\right) \\
& =2 \alpha\left[k^{\prime}\left(\frac{c_{1}+\alpha c_{2}}{(1-\alpha) \alpha}-r\right)-k^{\prime}\left(-\frac{c_{2}}{\alpha}\right)\right]
\end{aligned}
$$

which is positive when $c_{2} \rightarrow 0$.

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[^0]:    कthis paper is supported by National Natural Science Foundation of China (grant nos. 71922021 and 71773131), Shanghai Pujiang Program (16PJC035), and Beijing municipal fund for building world-class universities (disciplines) of Renmin University of China. We are grateful for valuable comments from two anonymous reviewers and the Editor.

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    https://doi.org/10.1016/j.geb.2021.01.009
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[^1]:    ${ }^{1}$ Branco et al. (2012) and Ke et al. (2016) include firms' price decision in their models, but only consider monopoly pricing.
    2 For technical convenience, we assume each consumer's outside option is $-\infty$ so that she always ends up purchasing one unit of the product.

[^2]:    ${ }^{3}$ While the assumption that $f$ and $g$ have infinite support simplifies our analysis, our main results do not depend on this assumption.
    4 We suppress any consumer-specific subscripts whenever there is no confusion.

[^3]:    which, according to equations (6) and (7), implies that $\hat{\varepsilon}_{l}=-\hat{\varepsilon}_{h}$.
    ${ }^{6}$ If the second-sample search cost is too high, that is, $c_{2}>\int_{0}^{\infty} \operatorname{tdF}(t)$, then it must be that $\hat{\varepsilon}_{h}<0<\hat{\varepsilon}_{l}$. In this case, if the current option $x$ is positive, the consumer will obtain $x$ immediately; if $x$ is negative, the consumer will blind buy the next product. Thus, searches never take place in stage 2 . One can also see from the proof of Proposition 2 that, $c_{1}<\int_{0}^{\infty} t d F(t)$ ensures that the search ever takes place in stage 1.
    ${ }^{7}$ In a duopoly setting, eliminating one product means the consumer will ultimately purchase the rival product. In other words, an elimination suffices to result in a purchase decision and ends the search process. However, for more than two firms, the search problem becomes significantly complex since the optimal search order is history dependent in general (see Doval, 2018). Hence, we restrict our analysis to the duopoly setting for a tractable solution.

[^4]:    ${ }^{8}$ We thank an anonymous referee for providing this intuitive explanation.
    ${ }^{9}$ Lemma 1 relies on symmetry of the distribution of $\tilde{\varepsilon}$. Generally, allowing for asymmetric distribution of $\tilde{\varepsilon}$ significantly complicates the sequential search problem and does not permit a tractable solution. To complement our analyses and to highlight the impact of blind buying to the optimal search order, we relax the assumption that $\tilde{\varepsilon}$ is symmetrically distributed in Section 4 . However, for tractability, we assume that $\tilde{\varepsilon}$ has a binary distribution (which need not be symmetric). In Section 4.2, we resolve the market equilibrium and show that under asymmetric $\tilde{\varepsilon}$, consumers may first sample the product with a lower net prior value. This is in sharp contrast to the Pandora's rule of Weitzman (1979).

[^5]:    10 Firm $i$ 's demand would be zero if $\tilde{\varepsilon}_{i} \leq-\hat{\varepsilon}_{h}-\Delta_{i}$, in which case the consumer will blindly buy product $j$.

[^6]:    11 Note that although the search cost in our model is stage-specific instead of seller-specific, the Pandora's rule is still applicable. The reason is that the consumer always inspects a product in stage 1 so that the first-sample search cost $c_{1}$ does not affect the search behavior. Thus, the optimal search strategy remains unchanged when $c_{1}=c_{2}$, in which case the search cost becomes seller-specific, as is assumed in the Pandora's rule.

[^7]:    12 Our intuition is in line with the results of Armstrong and Zhou (2020), who point out that possessing little information results in poor consumer and product match, but leads to fierce price competition.
    13 Without "blind buying", the first-sample search cost $c_{1}$ has no impact on the market equilibrium since the consumer's outside option is $-\infty$.

[^8]:    14 The formal proof is available upon request.
    15 Recall that the net prior value $v_{i}=\eta_{i}-p_{i}$, for $i=1,2$.

[^9]:    ${ }^{16}$ This subsection is a special case of section 2 when and only when $\alpha=\frac{1}{2}$.
    17 Here, we define search prominence as being sampled first, not being bought blindly first.

