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Contests with endogenous discrimination

Sanxi Li [a,*], Jun Yu [b,*]

[a] School of Economics, Renmin University of China, Beijing 100872, China
[b] Department of Economics, Northwestern University Evanston, IL, 60208-2600, United States

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ABSTRACT

This paper builds a complete information contest model with endogenous discrimination. We show that a revenue-maximizing contest designer will optimally set a bias towards a weaker contestant against a stronger contestant and completely eliminate the asymmetry between the two. Moreover, in contrast to fair contest models, where the revenue-maximizing contest designer is better off if the weaker contestant becomes stronger or the stronger contestant becomes weaker, our model shows that the opposite result may arise.

1. Introduction

Consider a complete information contest, modeled as an all-pay auction, with two contestants competing for one prize. Standard fair contest models (e.g. Hillman and Riley, 1989) predict that a selfish, revenue-maximizing contest designer is better off when the weaker contestant becomes stronger, while the designer becomes worse off when the stronger contestant becomes stronger. The result is intuitive by considering the contestants' equilibrium strategies. In equilibrium, conditional upon entry, both contestants bid uniformly from zero to the weaker contestant's valuation. The stronger contestant enters with probability 1, while the weaker contestant only enters with a probability strictly smaller than 1. When the weaker contestant becomes stronger, the contest designer enjoys two benefits. First, conditional upon entry, both contestants bid more aggressively. Second, the weaker contestant enters more often. However, when the stronger contestant becomes stronger, the contest designer suffers. Conditional upon entry, both contestants' bidding strategies remain the same: they still bid uniformly from zero to the weaker contestant's valuation. However, the weaker contestant enters less often, which hurts the contest designer.

In practice, contest designers often discriminate. For example, in public hiring and contracting, a bureaucrat/regulator is responsible for determining and announcing specific requirements for contestants. By manipulating those requirements, the bureaucrat/regulator can give some candidates known advantages over others, an abuse of the contest designer's power for which clear empirical evidence exists (Epstein et al., 2011).

In this paper, we consider a simple contest model, which allows the contest designer to be able to optimally control discrimination among contestants. We show that while allocative efficiency requires that the stronger contestant be favored to the greatest extent possible, the revenue-maximizing contest designer will set a bias towards the weaker contestant, which completely eliminates the asymmetry between the two contestants. This result is consistent with Clark and Riis (2000), who also consider an unfair contest, but with incomplete information.

Our model yields different predictions than does a fair model. Specifically, the contest designer obtains a higher expected revenue as the stronger contestant becomes stronger. Whether the contest generates higher revenues as the weaker contestant becomes stronger is ambiguous: while this is the case if the asymmetry between the two contestants is small, the result can be reversed if the asymmetry between the two contestants is large.

2. The model

There are two risk-neutral contestants competing for one prize through a contest, which is modeled as an all-pay auction. The prize valuations of the contestants are denoted by v_i, with $v_1 > v_2$. All the prize valuations are public information. Denote by $p_i(b_1, b_2)$ the winning probability of contestant i given their bids b_1 and b_2, with $b_i \geq 0$. When $b_i = 0$, we say that the contestant i does not

* Corresponding authors.
E-mail addresses: lisanxi@gmail.com (S. Li), junyu2008@u.northwestern.edu (J. Yu).

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enter the contest. The expected net payoff of contestant i is

$$Eu_i = p_i(b_1, b_2)\, v_i - b_i, \quad (i = 1, 2).$$

The function $p_i(b_1, b_2)$ is given by

$$p_1(b_1, b_2) = \begin{cases} 1, & \text{if } b_1 + r > b_2 \\ \frac{1}{2}, & \text{if } b_1 + r = b_2 \\ 0, & \text{if } b_1 + r < b_2, \end{cases} \quad (1)$$

where the discrimination variable r is selected by the contest designer. That is, the contestant with a higher effective bid wins the prize, and the prize is randomly awarded to the contestants in the event of a tie. Without loss of generality, we assume that contestant 1's effective bid is the sum of his actual bid b_1 and the contest designer's bias r, while contestant 2's effective bid is equal to his actual bid.

Note that although the contest designer's bias is added on contestant 1's side, the winning rule given in (1) does not necessarily favor contestant 1. Specifically, the bias is in favor of contestant 1 only when $r > 0$, while $r < 0$ implies that contestant 2 is favored. For $r = 0$, the contest is fair with no discrimination.

The contest designer cares only about the expected revenue that he can get from the two contestants:

$$E\pi = E(b_1 + b_2).$$

The seminal paper by Hillman and Riley (1989) provides a foundation for analyzing fair contest models. We summarize their findings in the following proposition.

Proposition 1 (*Hillman and Riley*). *In a fair contest, there exists a unique mixed strategy equilibrium. Contestant 1 always enters the contest while contestant 2 enters with probability $\frac{v_2}{v_1}$. Conditional upon entry, each contestant bids according to a uniform mixed strategy over the interval $[0, v_2]$.*

Given the contestants' equilibrium strategies, it is easy to see how the contest designer's revenues change with contestant valuations. The contest designer benefits from an increase in v_2: first, and, conditional upon entry, both contestants bid more aggressively in the sense that the upper bound of their uniform mixed strategies increases; second, contestant 2 enters more often. However, an increase in v_1 hurts the contest designer. The bidding strategies of contestants, conditional upon entry, remain unchanged, while contestant 2 enters less often.

Corollary 1. *The contest designer's expected revenue is decreasing in v_1 and increasing in v_2.*

The corollary states that the contest designer is worse off as the asymmetry between the two contestants increases. Both the weak contestant becoming weaker and the strong contestant becoming stronger will unambiguously hurt the contest designer. In the following section, however, we will show that this is not true as long as the contest designer can optimally control discrimination between the two contestants.

3. Optimal discrimination

First, we derive the equilibrium strategies for a given bias r. Before any analysis, it is obvious that $r \geq v_2$ or $r \leq -v_1$ is not optimal for the contest designer. For $r \geq v_2$, contestant 1 wins with probability 1 by bidding zero, since contestant 2 never bids over v_2; similarly, for $r \leq -v_1$, contestant 2 wins with probability 1 by bidding zero. Hence, we can restrict our attention to $r \in (-v_1, v_2)$.

In the following analysis, we focus on the case of $r \in (-v_1, 0)$. That is, the bias is in favor of the weak contestant 2. The analysis is the same for the case where $r \in (0, v_2)$. The equilibrium is described as follows.

Lemma 1. (i) *If $-v_1 < r \leq -\Delta$, contestant 1 always bids positively, while contestant 2 bids positively with probability $\frac{v_2}{v_1}$. Conditional upon bidding positively, contestant 1 bids uniformly over the interval $[-r, -r + v_2]$, while contestant 2 bids uniformly over the interval $[0, v_2]$.*

(ii) *If $-v_1 < r \leq -\Delta$, contestant 1 bids positively with probability $\frac{v_1+r}{v_2}$, while contestant 2 bids positively with probability $\frac{v_1+r}{v_1}$. Conditional upon bidding positively, contestant 1 bids uniformly over the interval $[-r, v_1]$, while contestant 2 bids uniformly over the interval $[0, v_1 + r]$.*

Proof. The proof is similar to Hillman and Riley (1989), and is omitted here.[1] □

The above lemma allows us to get the contest designer's revenue:

$$E\pi = \begin{cases} \frac{1}{2}\left(v_2 + \frac{v_2^2}{v_1}\right) - r & \text{for } -\Delta < r \leq 0 \\ \frac{1}{2}\left(\frac{(v_1 + r)^2}{v_1} + \frac{v_1^2 - r^2}{v_2}\right) & \text{for } -v_1 < r \leq -\Delta. \end{cases} \quad (2)$$

It follows from (2) that, for $r \leq 0$, the contest designer's revenue is maximized when $r = -\Delta$. Adopting a similar analysis, we can obtain the equilibrium strategies and therefore the contest designer's revenue for the case when $r > 0$. After careful comparison, we find that $r = -\Delta$ is optimal for the contest designer among all the values of bias r.

Proposition 2. *The optimal value of discrimination is $r^* = -\Delta$, where $\Delta = v_1 - v_2$ represents the valuation difference between the two contestants. Given optimal discrimination, contestant 1 always enters the contest while contestant 2 enters with probability $\frac{v_2}{v_1}$. Conditional upon entry, contestant 1 bids uniformly over the interval $[\Delta, \Delta + v_2]$, while contestant 2 bids uniformly over the interval $[0, v_2]$.*

Intuitively, favoring the weaker contestant has a positive effect on the contest designer's revenue, as long as the stronger contestant is still advantageous over his rival. On the one hand, the weak contestant's bidding strategy is not affected by the favorable discrimination (see Lemma 1). On the other hand, the stronger contestant, with a decreasing advantage over his rival due to the discrimination, has to bid more aggressively in order to win. Additionally, Lemma 1 also suggests that this positive effect vanishes as the discrimination becomes so favorable that the weak contestant becomes advantaged over his rival. This is because the more the weaker contestant is favored, the more likely that he wins without competition, which generates no revenue to the contest designer. Thus, the optimal amount of discrimination is achieved by just eliminating the advantage of the stronger contestant and creating equality between the two contestants.

Under the optimal bias $r^* = -\Delta$, the contest designer's expected revenue is then

$$E\pi = \frac{v_2^2}{2v_1} + v_1 - \frac{v_2}{2}, \quad (3)$$

from which we can immediately get the following corollary.

Corollary 2. *The contest designer's expected revenue is increasing in v_1. Moreover, it is non-monotonic in v_2: it decreases with v_2 when $\frac{v_2}{v_1} \leq \frac{1}{2}$ and increases with v_2 when $\frac{v_2}{v_1} \geq \frac{1}{2}$.*

[1] Detailed arguments are available upon request.

To understand the intuition, we decompose the contest designer's expected revenue in the following way:

$$E\pi = Eb_1 + Eb_2 = \frac{\Delta + (\Delta + v_2)}{2} + \frac{v_2}{v_1}\frac{v_2}{2}.$$

An increase in v_1 has two opposing effects on the contest designer's revenue. First, it makes contestant 1 bid more aggressively and hence increases the contest designer's profit. This is due to the fact that the contest designer optimally increases the bias in favor of contestant 2. This result contrasts with the case of a fair contest, under which increasing v_1 has no effect on contestant 1's bidding strategy. Second, increasing v_1 also deters the entry of contestant 2 and therefore reduces the contest designer's profit. The first effect unambiguously dominates the second one and hence the contest designer is better off.

An increase in v_2 also has two opposing effects on the contest designer's revenue. On the one hand, it reduces the extent of the optimal discrimination Δ. As a result, contestant 1 bids less aggressively: $\frac{\partial Eb_1}{\partial v_2} < 0$. On the other hand, it increases contestant 2's entrance probability $\frac{v_2}{v_1}$ and, conditional upon entry, makes contestant 2 bid more aggressively. When the asymmetry between the two contestants is large, the first effect dominates and hence the contest designer is worse off; when the asymmetry between the two contestants is small, the second effect dominates and hence the contest designer is better off.

4. Additive model vs. multiplicative model

Another way to model discrimination is with a multiplicative model (Epstein et al., 2011; Konrad, 2002[2]), in which the winning function is given by

$$p_1(b_1, b_2) = \begin{cases} 1, & \text{if } \delta b_1 > b_2 \\ \frac{1}{2}, & \text{if } \delta b_1 = b_2 \\ 0, & \text{if } \delta b_1 < b_2, \end{cases}$$

where $\delta > 0$ represents the discrimination factor and is controlled by the contest designer. The discrimination factor enters the winning function in an additive way in our model in previous sections, while it enters the winning function in a multiplicative way in the current model. In this section, we will show that, compared to the multiplicative model, the additive model generates not only a higher expected revenue for the contest designer, but also higher social welfare, defined as the sum of contestants' utilities as well as the contest designer's revenue.

Proposition 3 (*Epstein et al.*)**.** *The optimal discrimination factor is* $\delta^* = \frac{v_2}{v_1}$. *In equilibrium, both contestants enter with probability* 1. *Contestant* 1 *bids uniformly on* $[0, v_1]$*, while contestant* 2 *bids uniformly on* $[0, v_2]$.

As in the additive model, the optimal discrimination factor eliminates the asymmetry between the two contestants and creates equality between them. For the contest designer, relative to the additive model, the advantage of the multiplicative model is that contestant 2 enters more often; the disadvantage is that contestant 1 bids less aggressively, uniformly on $[\Delta, v_1]$ in the additive model and uniformly on $[0, v_1]$ in the multiplicative model.

[2] Konrad (2002) studies both multiplicative and additive discrimination. However, he does not consider optimal discrimination. Instead, discrimination is exogenous in his model.

It is easy to calculate the expected revenue of the contest designer in the multiplicative model:

$$E\pi = Eb_1 + Eb_2 = \frac{v_1 + v_2}{2}. \quad (4)$$

Using subscript m to denote the multiplicative model and a to denote the additive model, from (3) and (4), we have

$$E\pi_a - E\pi_m = \frac{(v_1 - v_2)^2}{2v_1} > 0.$$

Hence, the disadvantage of the multiplicative model always outweighs its advantage and therefore generates a lower expected revenue to the contest designer.

Now consider social welfare. Bids are just transferred from the contestants to the contest designer and hence do not affect social welfare. Social welfare depends only on efficiency, i.e., the average valuation of the winner in the contest. In the multiplicative model, from the equilibrium strategies, it is obvious that each contestant wins with probability 1/2. In the additive model, conditional upon entry, each contestant wins with probability 1/2. However, contestant 2 will remain inactive with a strictly positive probability. Thus, ex ante, contestant 1 wins with a probability strictly larger than 1/2. Compared to the multiplicative model, the additive model allocates the prize to the stronger contestant with a higher probability and hence is more efficient.

Proposition 4. *Compared to the additive model, the multiplicative model is less efficient and generates a lower expected revenue to the contest designer.*

5. Conclusion

We study optimal discrimination in an all-pay auction with complete information. The optimal bias completely eliminates the asymmetry between the two contestants. Moreover, unlike in a fair contest model, the contest designer here prefers the stronger contestant to be stronger. He prefers the weaker contestant to be stronger if the weaker contestant is not too weak, while the result is reversed if the weaker contestant is sufficiently weak. We also show that multiplicative discrimination is less efficient than additive discrimination.

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