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Can specialization be optimal when tasks are complementary? ☆

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ABSTRACT

Balmaceda (2016) shows that, under certain conditions, specialization can dominate multitasking when tasks are complementary, because specialization allows for a more flexible implementation of effort profiles than multitasking does. We show that this result is vacuously true, and multitasking always dominates specialization with task complementarity.

1. Introduction

Balmaceda (2016), henceforth B16, compares multitasking and specialization in a principal-agent model with moral hazard and limited liability, assuming that agents are compensated according to an aggregated performance measure. Its main contribution is the introduction of a trade-off that has been overlooked in the literature: when tasks are complementary, multitasking lowers the limited-liability rents that the principal has to pay to the agents, but specialization allows for a more flexible implementation of effort profiles.

The main result of B16 can be summarized as Proposition 1, which states that when tasks are complementary, there exist situations where the benefits from the flexible implementation of effort profiles dominate the costs of higher limited-liability rents; this leads the principal to prefer specialization to multitasking. Such a result is surprising and novel, because the literature suggests that substitution/conflict between tasks plays an important role in specialization (Holmström and Milgrom, 1991; Dewatripont and Tirole, 1999).

Unfortunately, we show that the aforementioned result is vacuously true, so that the trade-off highlighted in B16 actually does not actually occur. The intuition behind our finding is that when it is optimal for the principal to induce high efforts in at least one task under specialization, multitasking is always a better choice for him/her if the tasks are complementary.

The rest of the paper proceeds as follows. In section 2 we introduce Proposition 1 in B16. In section 3, we first show that part (ii) of Proposition 1 is vacuously true, and then prove that multitasking is always superior to specialization, as long as the tasks are complementary.

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2. Balmaceda's optimal task assignments

Suppose that a risk-neutral principal wants to delegate n tasks to agents. Two types of assignments are considered: Assignment (i) indicates full multitasking: the principal hires one agent who is responsible for all tasks; assignment (ii) assumes full specialization: the principal hires n risk-neutral agents and delegates each task to one agent. An agent's effort level in each task is either 0 (low) or 1 (high), and high effort incurs cost c while low effort incurs zero cost. The outcome space also has only two states: success and failure. Success brings a value of v while failure brings nothing. The probability of success is simply assumed to be given by

$$p(e) = p(\sum_{i=1}^{n} e_i).$$

Following B16, define $\triangle(k) \equiv p(k) - p(k-1)$, $B_k \equiv \frac{c}{p(k)-p(k-1)}$ and $W_k^s \equiv p(k)kB_k = \frac{p(k)}{\triangle(k)}ck$, for $k \in \{1, 2, ..., n\}$. Among these definitions, B_k measures the least bonus the principal has to pay in order to incentivize the kth agent to work hard. The term W_k^s, derived by aggregating every bonus, refers to the least total wage he/she has to pay to encourage k workers to make efforts in the case of specialization. The three types of technological relationships among tasks—strict complementarity (SSPM), strict substitutability (SSBM) and independence (IND)—are depicted by $\triangle(k+1) > \triangle(k)$, $\triangle(k+1) < \triangle(k)$ and $\triangle(k+1) = \triangle(k)$, respectively.

Before proposing Proposition 1 in B16, we find it helpful to first introduce Lemma 4, Lemma 5 and Lemma 7.

Lemma 4 *(B16). In specialization, suppose the following condition (CPs) holds*

$$\frac{W_k^s}{W_n^s} > \frac{p(k) - p(0)}{p(n) - p(0)}, \forall k \in \{0, ..., n-1\} \tag{CPs}$$

Then there exists a cutoff

$$v_{0n}^s \equiv \frac{cn}{p(n) - p(n-1)} \frac{p(n)}{p(n) - p(0)}$$

such that for all $v > v_{0n}^s$, the principal induces high effort in each task; for $v < v_{0n}^s$, he/she induces low effort in each task; and for $v = v_{0n}^s$, he/she is indifferent between inducing high effort in each task or no effort.

Lemma 5 *(B16). In specialization, suppose the following condition (NPs) holds*

$$\frac{W_{k+1}^s - W_k^s}{W_k^s - W_{k-1}^s} > \frac{p(k+1) - p(k)}{p(k) - p(k-1)}. \tag{NPs}$$

Then for each $k \in \{0, ..., n-1\}$, there exists cutoffs $v_{k,k-1}^s$ and $v_{k+1,k}^s$ such that for all $v_{k+1,k}^s > v \geq v_{k,k-1}^s$, the principal chooses to induce high effort in k tasks.

Lemma 7 *(B16). In multitasking, suppose that tasks satisfy SSPM. Then there exists a cutoff*

$$v_{0n}^{mc} \equiv \frac{cn}{p(n) - p(0)} \frac{p(n)}{p(n) - p(0)}$$

such that for all $v > v_{0n}^{mc}$, the principal chooses to induce high effort in each task; for $v < v_{0n}^{mc}$, he/she chooses to induce low effort in each task; and for $v = v_{0n}^{mc}$, he/she is indifferent between inducing high effort or no effort.

Lemma 4 and 5 introduce two requirements in addition to SSPM—conditions (CPs) and (NPs)—into technology $p(e)$. B16 shows that, (i) when SSPM and condition (CPs) hold, the optimal contract under specialization ends up inducing either all or none of the agents to work hard; (ii) when SSPM and condition (NPs) hold, the optimal contract under specialization can induce any effort profile. Lemma 7 suggests that, when $p(e)$ satisfies SSPM, the optimal contract under multitasking incentivizes the agent to work hard either in all or none of the tasks, depending on the parameter v. Combining Lemma 5 and Lemma 7, we can infer that when tasks are complementary and condition (NPs) holds, specialization allows for a more flexible implementation of the effort profiles than multitasking does. Thus, Proposition 1 in B16 is given as follows.

Proposition 1 *(B16). Suppose agents are compensated according to an aggregated performance measure and $p(e)$ satisfies SSPM. Then,*

i) *Suppose condition (CPs) holds. Then, for all $v \geq v_{0n}^{mc}$, the optimal assignment is multitasking and the principal implements the effort profile $(1, ..., 1)$, while for all $v < v_{0n}^{mc}$, the principal is indifferent between multitasking and specialization and he/she implements the effort profile $(0, ..., 0)$.*

ii) *Suppose condition (NPs) holds. Then, for all* $v \geq v_{0n}^{mc}$*, the optimal assignment is multitasking and the principal implements the effort profile n, while for all* $v < v_{0n}^{mc}$*, the optimal task assignment is specialization and the principal implements the effort profile in which effort is exerted in k tasks, with* $k \leq n-1$*, if v is such that* $v_{k+1,k}^{s} > v \geq v_{k,k-1}^{s}$.

A counterintuitive finding arises in part ii) of Proposition 1: when condition (NPs) and $v < v_{0n}^{mc}$ hold, specialization is the optimal task assignment for strict complementary tasks. According to B16, the reason is that when the value of success is sufficiently low, i.e., $v < v_{0n}^{mc}$, the optimal contract under multitasking induces low effort in all the tasks. On the other hand, since condition (NPs) holds, the optimal contract under specialization could incentivize some agents to work hard. As a result, specialization could generate a higher profit for the principal than multitasking does.

3. Multitasking dominates specialization

In this section, we prove that multitasking always dominates specialization. In subsection 3.1 we discuss the simplest case with two tasks to illustrate the key issue and intuition. Then, in subsection 3.2, we analyze a general case with more than two tasks.

3.1. A two-task example

Consider a two-task case as a simple example. The SSPM in this case is defined as $\triangle(2) \equiv p(2) - p(1) > p(1) - p(0) \equiv \triangle(1)$.

We first consider the case of specialization. It is obvious that any optimal contract that implements either of the effort profiles $(0,0)$ or $(1,1)$, can never make specialization dominate multitasking. Thus, we focus on the case where the optimal contract induces the effort profile $(1,0)$. That is, only one agent is incentivized to work hard.

Suppose agent 1 chooses high effort and agent 2 chooses low effort. The principal should set the bonus of agent 2 as $\beta_2^s = 0$, and choose β_1^s as

$$\beta_1^s = B_1^s \equiv \frac{c}{p(1) - p(0)} \tag{1}$$

The principal's maximized profit under specialization is given by

$$\pi_1^s = p(1)\left(v - B_1^s\right). \tag{2}$$

Note that for π_1^s to be nonnegative, we must have $v \geq B_1^s$.

Now consider the case of multitasking, where the principal assigns all tasks to one agent. Suppose the principal chooses a bonus β to induce high effort in both tasks. Then, β must satisfy the following incentive compatible constraint

$$p(2)\beta - 2c \geq \max\{p(0)\beta, p(1)\beta - c\}. \tag{3}$$

Due to $\triangle(2) > \triangle(1)$, equation (3) is equivalent to

$$\beta \geq \frac{2c}{p(2) - p(0)} \equiv B_2^m. \tag{4}$$

Notice that $B_2^m < B_1^s$ since $p(2) - p(1) > p(1) - p(0)$.

Thus, under multitasking, by implementing the profile $(1,1)$, the principal can obtain a profit of

$$\pi_2^m = p(2)\left(v - B_2^m\right). \tag{5}$$

Using (2), (5) and the fact that $B_2^m < B_1^s$, we have

$$\begin{aligned}
\pi_2^m - \pi_1^s &= p(2)\left(v - B_2^m\right) - p(1)\left(v - B_1^s\right) \\
&> p(2)\left(v - B_1^s\right) - p(1)\left(v - B_1^s\right) \\
&= [p(2) - p(1)]\left(v - B_1^s\right) \geq 0.
\end{aligned}$$

The example above shows that, even when the optimal contract for specialization implements the effort profile $(0,1)$, multitasking is still preferred to specialization. This implies that, in the two-task case, multitasking is always superior to specialization, as long as SSPM holds.

Our example contradicts part ii) of Proposition 1 in B16, which states that there exist situations where inducing one agent to exert effort in one task under specialization is better than multitasking for the principal. The key issue is that the specific condition raised in Proposition 1 can never hold. In our example, we find that, when it is optimal for the principal to induce one high effort under specialization, multitasking is always a better choice for him/her. This insight can be generalized because multitasking can internalize the positive externality of working hard in other complementary tasks as well as reduce the limited-liability rent the principal has to pay by enabling him/her to employ fewer agents; both of these can encourage the principal to adopt multitasking. In the next subsection, we provide a rigorous proof for this intuition.

3.2. The general case

This section analyzes a general case with more than two tasks. We first prove that part (ii) of Proposition 1 is vacuously true, and then show that multitasking is always superior to specialization as long as the tasks are complementary.

Claim 1. *Suppose $p(e)$ satisfies SSPM and condition (NPs) holds. As long as $v < v_{0n}^{mc}$, there does not exist any $k > 0$ such that $v_{k+1,k}^s > v \geq v_{k,k-1}^s$.*

Before we prove the aforementioned claim, it is helpful to establish the following lemma.

Lemma 1. *When $p(e)$ satisfies SSPM, $\frac{kp(k)}{[p(k)-p(0)]^2}$ decreases with k.*

Proof. We have

$$\frac{kp(k)}{[p(k)-p(0)]^2} = \frac{k}{p(k)-p(0)} \frac{p(k)}{p(k)-p(0)} = \frac{1}{\frac{1}{k}\sum_{i=1}^{k} \Delta(i)} \frac{p(k)}{p(k)-p(0)}.$$

Due to SSPM, $\Delta(k)$ increases with k so that $\frac{1}{k}\sum_{i=1}^{k} \Delta(i)$ increases with k, which implies $\frac{1}{\frac{1}{k}\sum_{i=1}^{k} \Delta(i)}$ decreases with k. This immediately proves our lemma since $\frac{p(k)}{p(k)-p(0)}$ also decreases with k. □

We are now ready to prove Claim 1.

Proof for Claim 1. Suppose that given $v < v_{0n}^{mc}$, there exists some $k \in \{1, 2, ..., n\}$ satisfying $v_{k+1,k}^s > v \geq v_{k,k-1}^s$. Due to Lemma 5, the optimal contract under specialization incentivizes exactly k agents to work hard. Let β^s be the limited-liability rent required to induce high effort. The incentive compatible constraint gives that

$$p(k)\beta^s - c \geq p(k-1)\beta^s.$$

The optimal rent β^s should be the lowest value that satisfies the condition above, that is,

$$\beta^s = \frac{c}{p(k)-p(k-1)} \equiv B_k^s.$$

Thus, the optimal contract generates an expected profit of

$$\pi^s = p(k)\left(v - kB_k^s\right).$$

Note that the optimality of the contract implies that

$$p(k)\left(v - kB_k^s\right) > p(0)v, \tag{6}$$

where the right-hand side of the inequality represents the principal's profit when all agents choose $e = 0$.

The condition (6) implies

$$v \geq \frac{kp(k)B_k^s}{p(k)-p(0)}.$$

However, due to Lemma 1, we have

$$\frac{kp(k)B_k^s}{p(k)-p(0)} = \frac{ckp(k)}{[p(k)-p(0)][p(k)-p(k-1)]} \geq \frac{ckp(k)}{[p(k)-p(0)]^2} \geq \frac{cnp(n)}{[p(n)-p(0)]^2} = v_{0n}^{mc},$$

which contradicts the fact that $v < v_{0n}^{mc}$. □

Claim 1 states that, for strictly complementary tasks, if the optimal contract induces zero effort for all the tasks under multitasking, it should also induce zero effort under specialization. Thus, part (ii) of Proposition 1 in B16 is vacuously true because the set of conditions under which specialization is superior to multitasking is empty.

The logic behind Claim 1 is that, for strictly complementary tasks, as long as it is profitable for the principal to incentivize at least one agent to work hard under specialization, deviating to multitasking always makes him/her better off, regardless of whether conditions (CPs) and (NPs) are satisfied. This leads to our result regarding Proposition 1′, which implies that the trade-off highlighted in B16 actually does not exist. Specifically, in the case of specialization, there are no benefits of the flexible implementation of the effort profiles. The reason is that, while allowing for more flexible effort profiles, specialization also requires a higher level of output upon success in order to implement the effort profiles.

Proposition 1′. *Suppose agents are compensated according to an aggregated performance measure and $p(e)$ satisfies SSPM. Then, for any $v \geq 0$, the optimal assignment is always multitasking.*

Proof. It is understood that specialization is never the optimal assignment if it induces zero effort in all the tasks. Now suppose the optimal contract under specialization incentivizes k agents to work hard for some $k > 0$. According to Claim 1, we must have $v \geq v_{0n}^{mc}$ so that the optimal contract under multitasking implements the effort profile $(1, 1, ...1)$. Let $\pi^s(k)$ and π^m be the principal's maximized profits under specialization and multitasking, respectively.

According to the proof of Claim 1, we have

$$\begin{aligned}\pi^s(k) &= p(k)\left(v - kB_k^s\right)\\ &= p(k)\left[v - \frac{kc}{p(k) - p(k-1)}\right]\end{aligned}$$

and

$$\begin{aligned}\pi^m &= p(n)\left(v - B_{sspm}\right)\\ &= p(n)\left(v - \frac{nc}{p(n) - p(0)}\right)\end{aligned}$$

Thus, we have, for any $0 < k \leq n$,

$$\begin{aligned}\pi^m - \pi^s(k) &= v\left[p(n) - p(k)\right] - c\left[\frac{p(n)n}{p(n) - p(0)} - \frac{p(k)k}{p(k) - p(k-1)}\right]\\ &\geq v_{0n}^{mc}\left[p(n) - p(k)\right] - c\left[\frac{p(n)n}{p(n) - p(0)} - \frac{p(k)k}{p(k) - p(k-1)}\right]\\ &= \frac{cnp(n)\left[p(n) - p(k)\right]}{\left[p(n) - p(0)\right]^2} - c\left[\frac{p(n)n}{p(n) - p(0)} - \frac{p(k)k}{p(k) - p(k-1)}\right]\\ &= c\left[p(k) - p(0)\right]\left\{\frac{p(k)k}{\left[p(k) - p(k-1)\right]\left[p(k) - p(0)\right]} - \frac{p(n)n}{\left[p(n) - p(0)\right]^2}\right\}\\ &\geq c\left[p(k) - p(0)\right]\left\{\frac{p(k)k}{\left[p(k) - p(0)\right]^2} - \frac{p(n)n}{\left[p(n) - p(0)\right]^2}\right\} \geq 0,\end{aligned}$$

where the first inequality follows from the fact that $v \geq v_{0n}^{mc}$, and the last inequality follows from Lemma 1. This completes our proof. □

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