International Journal of Industrial Organization 87 (2023) 102924

Contents lists available at ScienceDirect

International Journal of Industrial Organization

journal homepage: www.elsevier.com/locate/ijio

Competitive targeted online advertising☆

Sanxi Li[a], Hailin Sun[b], Jun Yu[c,*]

[a] School of Economics, Renmin University of China; Center for Digital Economy Research, Renmin University of China; Research Institute of State-owned Economy, Renmin University of China, Beijing, 100872, China
[b] Institute for Social Governance and Development Research, Tsinghua University, 14F, #2 Shuangqing Building, 77 Shuangqing Road, Beijing, 100085, China
[c] School of Economics, Shanghai University of Finance and Economics; Key Laboratory of Mathematical Economics (SUFE), Ministry of Education, 777 Guoding Road, Shanghai 200433, China

A R T I C L E I N F O

Article history:
Received 4 August 2020
Revised 16 January 2023
Accepted 23 January 2023
Available online 25 January 2023

JEL classification:
D2
D4
L1
L2

Keywords:
Online shopping platforms
Targeted advertising
Position auction
Price discrimination

A B S T R A C T

This paper examines how an online publisher utilizes its information about consumer preference to target advertising. In our model, two firms first bid for a prominent ad position in a publisher-organized position auction, and then compete on price in the subsequent product marketplace. We consider two dimensions of consumer heterogeneity. First, consumers are heterogeneous in product preference. Based on their tastes, some consumers prefer one product over the other, whereas other consumers may rank the products in an opposite order. Second, consumers differ in search preference, i.e., “nonshoppers” only consider the advertised product, while “shoppers” always search both firms’ products before buying. We show that targeted advertising based on product preference will mitigate price competition in product markets as well as competition in position auctions, the latter to the detriment of the publisher. In contrast, targeted advertising based on search preference always benefits the publisher, as the winning firm can charge monopoly prices to nonshoppers. We show that the publisher’s optimal choice is to utilize only the information about consumer search preference. We also explore the welfare implications of targeted advertising based on different types of consumer preference.

1. Introduction

Search advertising, also known as a position auction, allows advertisers to submit bids to online publishers on specific keywords in order for their ads to appear prominently in the search results. Online publishers use a combination of submitted bids and ad relevance to rank the ads. Search advertising is considered as one of the most effective forms of online advertising because it is close to the user’s buying decision and matches it to the user’s stated information needs. Online publishers, such as search engines, social media and e-commerce marketplaces have succeeded in generating significant revenues from search ads. According to the financial reports of these companies, Google, Facebook, and Amazon generated

☆ We are grateful for valuable comments from the Editor Jose L. Moraga-Gonzalez, two anonymous reviewers, Yijuan Chen and participants in seminars at Southwestern University of Finance and Economics and Fudan University. Li thanks the financial support by National Natural Science Foundation of China (grant nos. 71773131; 71922021; 72192801), Beijing Natural Science Foundation (Z220001), and fund for building world-class universities (disciplines) of Renmin University of China; Yu thanks the financial support by National Natural Science Foundation of China (grant no. 72273078), and support from the SUFE Theoretical Economics Gaofeng II Discipline Innovation Project (2018110721).

* Corresponding author.
E-mail addresses: sanxi@ruc.edu.cn (S. Li), sunhailin@mail.tsinghua.edu.cn (H. Sun), yu.jun@mail.shufe.edu.cn (J. Yu).

https://doi.org/10.1016/j.ijindorg.2023.102924

advertising revenues of $209.49 billion, $115 billion, and $31.16 billion respectively in 2021, with paid search ads comprising the bulk of this revenue.

Given the success of this business model in practice, a range of related literature has theoretically investigated the bidding strategies of advertisers and the revenues of online publishers, as well as the impact of online advertising on consumer surplus and social welfare (Varian, 2007; Chen and He, 2011; Athey and Ellison, 2011; Xu et al., 2011). Most studies have considered uniform search advertising, in which all consumers receive the same ad given by a position auction. However, with the rise of big data and artificial intelligence, online publishers have been able to collect and process unprecedented amounts of consumer-level data, such as consumers' browsing behavior, their profiles on company sites, shopping habits and other basic demographic information, which online publishers can use to tailor ads to consumers' interests and specific traits.[1] Targeted advertising is important as it increases brand awareness and the engagement rate with consumers. For example, Google allows all advertisers to run interest-based ads on its display network (Bazilian, 2011). Advertisers on Facebook can use consumers' demographic information, social activities on the platform, as well as social networks to target their potential customer base.[2] Amazon's sponsored ads can also be targeted to individual products, product categories or buyer interest.[3]

This paper incorporates targeted advertising into the traditional position auction model and examines whether and how online publishers should use consumer data for targeted advertising. We also examine how targeted advertising changes firms' bidding strategies in position auctions and their pricing strategies in product markets, and whether targeted advertising has a positive impact on welfare.

The existence of consumer heterogeneity is crucial for online publishers to develop targeted advertising. This paper considers two types of heterogeneity. First, consumers are heterogeneous in product preference. Based on their tastes, some consumers prefer one product over the other, whereas other consumers may rank the products in an opposite order. Second, consumers differ in search preference, i.e., some are experienced online shoppers, who prefer to surf and find the best deals (Childers et al., 2001), while others may be constrained by time or skills and therefore search only a limited number of advertisers.

In some cases, data on online activity can only support online publishers in deriving a consumer's product preference. For example, an e-commerce marketplace can only track a user's product purchase history, i.e., which products he/she has purchased, but not his/her browsing behavior, i.e., how many products he/she searched for before making a final purchase decision.[4] Another example is that a consumer's geographical location derived from his/her IP address is useful for inferring their product preferences, e.g. people living in Sichuan province in China generally prefer spicy food, while those living in Jiangsu province prefer sweet food. However, this type of data provides little information about consumers' search preferences, as consumers in one province are unlikely to search more frequently than those in another province on a systematic basis. Thus, in this case, online publishers can target their ads based only on product preference (TP).

In other cases, online publishers can infer both product and search preferences. Cookies and user accounts profiles enable online publishers to track consumer browsing history,[5] time spent on each clicked content, and search habits while shopping, allowing publishers to accurately infer a consumer's search preference. In this case, online publishers can deliver targeted advertising based on product preference (TP), based on search preference (TS) or both (T2). Our study encompasses these two dimensions of consumer heterogeneity and discusses the effects of different targeting techniques.

In our model, we consider an online publisher, two firms A and B, and a continuum of consumers with measure one. The online publisher allows the two firms to compete for a prominent ad position through a second-price position auction. The product of the firm that wins the prominent ad position will be advertised and thus first noticed by each consumer. After the position auction, firms compete on price in the subsequent product market. Consumers differ in their product preferences. A type i consumer prefers product i to product j in the sense that product i is more likely to suit her taste than product j. We assume that firms are asymmetric, with firm A stronger than firm B, i.e., the proportion of type A consumers is higher than the proportion of type B consumers. Consumers also differ in their search preferences. Similar to Varian (1980), we assume that shoppers always search for both products, whereas nonshoppers search only for the product that is prominently displayed.

Targeting techniques allow the online publisher to divide the whole market into different submarkets, based on consumers' preference types. The online publisher can thus choose between uniform advertising (UA) and targeted advertising. Under UA, no targeting techniques are implemented and the prominent ad position is for the entire market. This means, the product of the firm that wins the position auction will appear prominently to all types of consumers. On the other hand, under targeted advertising, a position auction is organized in each submarket, which generates an advertised product (i.e. a prominent product) in that submarket. A firm may lose the position auction in one submarket, but win elsewhere. This means that the advertised products need not be the same for all the submarkets.

[1] By "targeting" we mean the act of distinguishing between different groups of consumers by an online publisher. The term has a completely different meaning in the marketing literature, where targeting is often considered a strategy by which sellers increase their market reach by providing relevant information to otherwise uninformed consumers.

[2] Please refer to https://www.facebook.com/policies/ads for details.

[3] Please refer to https://amzn.to/ebook-vu?resource=guide for details.

[4] This occurs when consumers delete cookies to hide their past browsing activities.

[5] A study by the Wall Street Journal (Angwin, 2010) shows that the top 50 websites in China use an average of 64 tracking technologies.

Existing literature on targeted advertising and targeted pricing focuses on the case where the online publisher can only identify consumers' product preferences and thus choose between UA and TP. In our model, the publisher has data on two dimensions of consumer preferences, and can choose among four types of targeting techniques: UA, TP, TS and T2. To the best of our knowledge, this situation has never been studied in the literature.

In the benchmark model, we focus on the case where firms can charge targeted prices based on targeted advertising, i.e., where firms charge different prices in different submarkets. Our results show that different targeting techniques have qualitatively different effects on final equilibrium and welfare. Compared to UA, TP changes the advertisements that appear prominently to consumers of type B, while TS does not change the advertisements to consumers. Under UA, the strong firm A appears prominently to all consumers. Under TP, each firm wins its strong market, i.e., firm i wins prominence to type i consumers, for $i = A, B$. Under TS, the strong firm A still wins prominence toward nonshoppers, while prominence toward shoppers is irrelevant, as they always search for both products.

TP generates two different effects on the publisher's auction revenue. On the one hand, the implementation of TP increases the asymmetry of the firms and thus reduces price competition in the product market, which increases firms' incentives to bid for the prominent ad position. On the other hand, the increase in asymmetry also reduced competition between firms in the position auction. TP benefits the online publisher if and only if the former effect dominates, which occurs when there are a sufficient number of shoppers. Because TP reduces price competition between firms in the product market, it increases industry profits and reduces consumer surplus. In addition, TP displays consumers' favorite products in their prominent advertising slots, which increases the likelihood of trade and overall social welfare.

The implementation of TS has a different effect on firms' pricing strategies in the product market and their bidding strategies in the position auction. In the position auction, the prominent position facing shoppers generates no value to firms as shoppers always search for both products. However, the prominent position in the submarket of nonshoppers generates a higher value because the prominent firm can charge monopoly prices and earn monopoly profits given that nonshoppers only visit the prominent position. We show that the increase in auction revenue from the nonshopper submarket outweighs the loss from the shopper submarket, so that TS always benefits online publishers. In the product market, TS intensifies firms' price competition for shoppers, but mitigates competition for nonshoppers. Intuitively, shoppers search more than nonshoppers, so firms have less monopoly power over shoppers. We show that the fall in the average price for shoppers outweighs the rise in that for nonshoppers, and therefore TS always reduces industry profits and increases consumer surplus. Furthermore, TS does not change the product displayed in the prominent position for each consumer, and therefore does not affect the likelihood of trade or overall social welfare.

Our results show that, compared to T2, the publisher's total auction revenue is higher under TS. This means that the publisher may not prefer to apply targeted advertising by using all the information available. Specifically, data on consumer product preference should never be used whenever data on consumer search preference is available. We compare T2 with TS by asking the question that whether the publisher should additionally conduct TP when TS has already been implemented. As mentioned before, the positive effect of applying TP is that price competition in the product market is moderated and therefore the valuation of the prominent position is increased for both firms. However, under TS, for the submarkets of shoppers, the position auctions always generate a zero revenue, regardless of whether TP is employed or not. Thus, the effect of additionally implementing TP lies only in the submarkets of nonshoppers. Note that TS has already reduced price competition in the submarkets of nonshoppers to the greatest extent – the prominent firms are able to charge monopoly prices in the product markets. Thus, with the presence of TS, the positive effect of TP is greatly weakened and therefore additionally implementing TP is detrimental to the online publisher.

We further examine the case where price discrimination is not allowed. Thus, consumers in all submarkets must receive the same price for each product. A somewhat surprising result is that, in the absence of price discrimination, TS has no effect on equilibrium and is arguably meaningless. Thus, TP is the only relevant targeting technique. The reason is that TS does not change the advertisements displayed to consumers and, therefore, the demand functions of the firms remain unchanged regardless of whether the publisher implements TS or not. Additionally, we show that, when price discrimination is prohibited, UA always generates higher auction payoffs to the publishers than TP does. This is because TP has a limited moderating effect on price competition in product markets in the absence of price discrimination. Our results show that in the absence of price discrimination, publishers do not implement targeted advertising in any form.

The rest of the paper is organized as follows. We review the relevant literature in Section 2 and present the formal model in Section 3. In Section 4, we address the market equilibrium for each targeting technique, based on which we derive firms' bidding strategies in the position auctions. We then compare the online publisher's auction revenues, equilibrium prices, industry profits and consumer surplus for different targeting techniques in Section 5. We examine the case where price discrimination is not permitted in Section 6, and discuss policy implications in Section 7. Section 8 provides extensions, and Section 9 concludes the paper. All omitted proofs are provided in the Appendix.

2. Literature review

This paper relates to the literature of search prominence, in which consumers follow a specific search order rather than searching randomly. Armstrong et al. (2009) were among the first to investigate how search prominence affects equilibrium prices and welfare.[6] Armstrong and Zhou (2011) and Haan and Moraga-Gonzalez (2011) explored sources of prominence, including advertising, commission payments, prices, repeat purchases, and position auctions. Traditional position auction models examine firms' bidding strategies and auctioneers' optimal design of auction rules, as well as consumers' subsequent search behavior and firms' pricing behavior in the post-auction product markets (Athey and Ellison, 2011; Chen and He, 2011; Anderson and Renault, 2015).[7] However, to the best of our knowledge, no studies in this literature have considered the possibility that publishers can collect data on consumer preferences and use this data to implement targeted prominence/position auctions. Our contribution to this stream of literature is our inclusion of targeted advertising and our exploration of the impact of targeting techniques on market behavior and welfare.

Our study also adds to the literature on targeted advertising. Most of this literature focuses on firms' choices of independent advertising expenditures in targeted advertising models without publishers/platforms and without position auctions. Iyer et al. (2005) showed that, as a means of differentiation, firms strategically choose to advertise less in weaker submarkets than in stronger submarkets. In contrast, Esteves and Resende (2016) examine firms' advertising strategies in the presence of price discrimination and show that firms are likely to advertise more intensively in weaker markets. Gal-Or and Gal-Or (2005) and Galeotti and Moraga-Gonzalez (2008) showed that targeting techniques can soften firms' price competition in the product market and increase their profits. Brahim et al. (2011) study targeted advertising under Hotelling competition without price discrimination and show that targeting techniques may reduce firms' equilibrium profits. Johnson (2013) investigated the effect of targeting techniques when consumers can make efforts to avoid advertising and showed that targeting techniques are beneficial to firms but harmful to consumers. They also demonstrated that price discrimination may increase as well as decrease firms' profits. Zhang and He (2019) studied targeted advertising strategies for asymmetric firms and show that targeting techniques may benefit high-cost firms at the expense of low-cost firms.

Our study of online advertising differs from the above literature in two respects. First, publishers have detailed consumer data and play a key role in online advertising by deciding whether and how to use targeting techniques. The interests of online publishers are generally different from those of firms. Publishers may choose not to use targeting techniques even if doing so would increase the firms' profits. In the above study, publishers' strategies were not considered, and we believe this is inadequate when examining online advertising. Second, in our analysis, online advertising is exclusive and determined by position auctions. This means that firms can no longer choose their advertising expenditures independently, i.e., if one firm wins the auction, the other loses prominence and the amount the winning firm has to pay for advertising is determined by the bids of the losing firm.

To the best of our knowledge, Chen and Stallaert (2014) is the only relevant paper that combines position auctions with targeted advertising. They considered a model in which the online publisher can target ads based on consumers' product preferences, with each consumer's click value being exogenous to the advertiser. They showed that when there are many advertisers, targeting techniques can increase auction revenue for online publishers. However, UA always generates higher auction revenue when there are only two advertisers. Our paper differs from theirs in two important respects. First, in their paper, advertisers' valuation of prominence is exogenously given, whereas we argue that the value of prominence is endogenously determined by firms' pricing behavior in subsequent product market. We show that TP generates higher auction revenues for the publisher even with only two advertisers, which contrasts with their results. Second, we also consider the possibility of targeting ads based on consumers' search preferences, which was not considered in their paper. The reason for studying TS is twofold: firstly, there is indeed considerable heterogeneity in consumer search behavior, and advances in information technology have enabled online publishers to infer these qualitative heterogeneities from online activity; secondly, as we will show in Section 5, TS has a qualitatively different impact on equilibrium outcomes compared to TP.

Finally, our paper is related to the literature on price discrimination based on competitive behavior (e.g., Villas-Boas, 1999; Fudenberg and Tirole, 2000; Fudenberg and Villas-Boas, 2006). A common finding is that firms typically face a prisoner's dilemma: all firms practice price discrimination and earn lower profits in equilibrium, even though they may be better off by cooperating and agreeing to a uniform pricing strategy. In contrast, our study shows that price discrimination combined with targeted advertising can mitigate price competition and increase industry profits, echoing the findings of two studies by Chen and Pearcy (2010) and Shin and Sudhir (2010).

3. The model

Two firms, A and B, sell products to a unit mass of consumers on an e-commerce marketplace. Production costs are normalized to zero. Firms first submit bids to an online publisher for a prominent advertising position. The position auction is modeled as a second-price auction, where the firm with the highest bid wins the prominent position and pays the second highest bid.[8] A prominent position can be interpreted as the top sponsored advertising slot listed on an e-commerce

[6] See also Wilson, 2010;Rhodes, 2011; Zhou, 2011; and Fishman and Lubensky, 2018; Moraga-Gonzalez et al. (2021).

[7] See also Edelman et al., 2007; Varian, 2007; Chen et al., 2009; Ghose and Yang, 2009; Katona and Sarvary, 2010; Liu et al., 2010; Jerath et al., 2011; Xu et al., 2011; Yao and Mela, 2011; Zhu and Wilbur, 2011; Desai et al. 2014; Liu and Viswanathan, 2014; Sayedi et al., 2014; Yang et al., 2014; Amaldoss et al., 2015; Lu et al., 2015; Shin, 2015; Yu et al., 2018.

[8] Alternatively, we can consider the following second-price "per-click" auction. Each firm submits a per-click bid. The result of the auction is based on a score that is equal to the per-click bid multiplied by the expected number of clicks for which the firm is placed in a prominent position. The firm with the

platform or search engine. After observing who wins the prominent position, firms compete for consumers by simultaneously setting the prices of their products in the subsequent product market.

Consumers have unitary demand and different preferences for the two products. We assume that a γ proportion of consumers, called type A consumers, have a relative preference for product A; while a $1-\gamma$ proportion of consumers, called type B consumers, have a relative preference for product B. Without loss of generality, we assume $\gamma \geq 1/2$ so that firm A is stronger than firm B. In what follows, we refer to the strong (weak) firm in the market as the high-type (low-type) firm.

Let v_i^j denote the valuation of product i for a type j consumer, satisfying the following conditions:

$$v_i^j = \begin{cases} 1 \text{ with probability } \alpha_{ij} \\ 0 \text{ with probability } 1-\alpha_{ij} \end{cases}.$$

With probability α_{ij}, product i matches the taste of a type j consumer and delivers a utility normalized to 1. With probability $1-\alpha_{ij}$, product i does not match the taste of a type j consumer and delivers zero utility.

We assume that $\alpha_{AA} = \alpha_{BB} = \alpha_h$, $\alpha_{AB} = \alpha_{BA} = \alpha_l$, and $1 > \alpha_h > \alpha_l > 0$. That is, product i has a higher probability of matching the taste of a type i consumers than product $j \neq i$, where $i, j \in \{A, B\}$. The matching probability of product A to a random consumer is equal to $\alpha_A = \gamma\alpha_h + (1-\gamma)\alpha_l$ and that of product B is equal to $\alpha_B = \gamma\alpha_l + (1-\gamma)\alpha_h$. Since $\gamma \geq 1/2$, we have $\alpha_A \geq \alpha_B$.

Consumers search for ad slot(s) to obtain price information and valuation(s) of the product(s). We assume that all consumers will first search for the prominent position. Similar to Varian (1980), consumers' search behavior is heterogeneous. β proportion of consumers are shoppers, who always search for both firms. $(1-\beta)$ proportion of consumers are nonshoppers, who only search for the prominent position. We assume that consumers' search preferences are independent of their product preferences.

The publisher has full or partial information on consumer types. There are four types of consumers: type A shoppers, type B shoppers, type A nonshoppers and type B nonshoppers. Thanks to information technology, online publishers are able to accurately infer consumer preferences. Existing literature on targeted advertising and targeted pricing focuses on cases where online publishers identify only the product preferences of consumers. However, recent developments in tracking technology have enabled online publishers to obtain detailed data on consumers' online activities, including browsing and search habits when shopping, so that publishers can also accurately infer consumers' search preferences. The case studied in this paper is that online publishers can target advertisements based on product preferences (TP), search preferences (TS) or both preferences (T2).

Based on the targeting techniques, the publisher can divide the whole market into different submarkets, according to consumers' preference types. It is thus possible to conduct a position auction in each submarket, which generates an advertised product in that submarket. As a result, consumers from different submarkets may see different ads in the prominent positions. Finally, the publisher may choose not to use targeting technology and implement a uniform position auction (UA) for the entire market, so that all consumers receive the same advertisements.

Since the publisher has the technology to display different advertisements in different submarkets, it is technically feasible for firms to charge different prices in different submarkets. Thus, in the basic model, we consider the case where firms can charge targeted prices based on targeted advertising. In Section 6, we will examine the case where price discrimination is not permitted, so that consumers in different submarkets receive the same price for each product.[9]

The timing of our model is as follows. In the first stage, the publisher announces the rules of the auction and firms decide on their bidding strategy. In the second stage, firms observe who wins the position auction and simultaneously set prices for their products. In the third stage, consumers search for position(s) and make purchase decisions.

Benchmark. First, let us examine a benchmark setting. Assume that all consumers are shoppers and have the same product preferences. Furthermore, the probability that product i matches a consumer's taste is α_i, $i = 1, 2$. Without loss of generality, we assume that $\alpha_1 \geq \alpha_2$, i.e., firm 1 (firm 2) is the high-type (low-type) firm.

We first derive the equilibrium prices in this case. All consumers are shoppers and search for both firms. A $\alpha_i\alpha_j$ proportion of consumers will find that both products match their preference, and buy from the firm with the lower price. A slightly lower price than its competitor helps a firm to attract this fraction of consumers, thereby increasing a discontinuous growth in demand and profit. For any (p_i, p_j), where $p_i \leq 1$ and $p_j \leq 1$, the demand of firm i is

$$D_i(p_i, p_j) = \begin{cases} \alpha_i \text{ if } p_i < p_j \\ \alpha_i\left(1-\frac{\alpha_j}{2}\right) \text{ if } p_i = p_j \\ \alpha_i\left(1-\alpha_j\right) \text{ if } p_i > p_j \end{cases}. \tag{1}$$

Due to the discontinuity of the demand function, there is no pure-strategy equilibrium. Using a technique similar to Narasimhan (1988), we can derive the mixed-strategy equilibrium. We use $F_i(p)$ to denote the probability that firm i will set its price below p as the cumulative distribution function of its mixed strategy.

highest score wins the prominent position and pays on a pay-per-visit basis, so the per-visit payment generates a score equal to the second highest score. Such a "per-click" auction is equivalent to the auction we are considering here.

[9] It is possible that firms may perceive online price discrimination as potentially risky. As an example, consider the backlash following Amazon's alleged attempts at price discrimination (Anderson and Simester, 2010).

Lemma 1. *Assume that all consumers are shoppers and the matching probability of product i for each consumer is α_i with $\alpha_1 \geq \alpha_2$. Then, $F_2(p)$ is continuous on $[1-\alpha_2, 1)$ with*

$$F_2(p) = \frac{1}{\alpha_2} - \frac{1-\alpha_2}{\alpha_2}\frac{1}{p}; \tag{2}$$

$F_1(p)$ is continuous on $[1-\alpha_2, 1)$ with

$$F_1(p) = \frac{1}{\alpha_1} - \frac{1-\alpha_2}{\alpha_1}\frac{1}{p}. \tag{3}$$

Moreover, $F_1(p)$ has a mass point at $p = 1$ with a measure of $1 - \frac{\alpha_2}{\alpha_1}$.

The expected profit of firm 1 is

$$\pi_1 = \alpha_1(1-\alpha_2), \tag{4}$$

and the expected profit of firm 2 is

$$\pi_2 = \alpha_2(1-\alpha_2). \tag{5}$$

The above lemma characterizes an asymmetric price equilibrium when two firms have different matching probabilities and all consumers are shoppers. Price dispersion arises in equilibrium for the same reasons as in Narasimhan (1988). In fact, the model is equivalent to that of Narasimhan's (1988) in that a $\alpha_i(1-\alpha_j)$ proportion of consumers are loyal to firm i and a $\alpha_i\alpha_j$ proportion of consumers buy from the lower-price firm. Firms will weigh the benefits of charging higher prices to their loyal consumers against the benefits of attracting switching consumers through lower prices.

From the expressions of π_1 and π_2, the high-type firm (i.e., firm 1) always benefits when its own matching probability increases ($\alpha_1 \uparrow$) or its competitor's matching probability decreases ($\alpha_2 \downarrow$). However, the low-type firm (i.e., firm 2) may or may not benefit when its own matching probability increases ($\alpha_2 \uparrow$). Specifically, firm 2 benefits (hurts) from a marginal increase in its own matching probability when its own matching probability is initially low (high). Similar results can be found in the standard duopoly model with vertical quality differentiation (Shaked and Sutton, 1982).

The intuition is that, increasing the matching probability for high-type firms not only leads to higher demands, but also eases price competition, ultimately benefiting high-type firms. However, an increase in the matching probability for low-type firms has two opposite effects on their profits. On the one hand, a higher matching probability also increases the low-type firm's demand, which is beneficial for the profits. On the other hand, as the low-type firm's matching probability increases, the price competition between the two firms becomes increased, which can lead to lower profits. The demand effect dominates when the initial matching probability for the low-type firm is low, while the effect regarding price competition dominates when the initial matching probability of the low-type firm is high. These results and intuitions are important for the analysis that follows.

4. Equilibrium

This section derives equilibrium pricing strategies for the product markets and bidding results for the position auctions with different advertising techniques.

4.1. Uniform advertising

In this subsection, we consider uniform advertising (UA): the publisher does not implement targeted advertising and conducts a uniform position auction for the entire market. We first examine the price competition between the two firms when the prominent position is exogenously given, and then derive the bidding strategies for the uniform position auction.

Recall that the matching probability of firm A with a random consumer is α_A and the probability of matching firm B with a random consumer is α_B. First, suppose firm A wins prominence. A shopper searching for both firms' products has a matching probability of α_A for product A and α_B for product B. A nonshopper searching only for product A has a matching probability of α_A. Equivalently, a nonshopper can be considered as a shopper searching for both products, but with a matching probability of 0 for product B. For any $p_A \leq 1$ and $p_B \leq 1$, the demands for the two firms' products are:

$$D_A(p_A, p_B) = \begin{cases} \alpha_A \text{ if } p_A < p_B \\ \alpha_A\left(1 - \frac{\beta\alpha_B}{2}\right) \text{ if } p_A = p_B \\ \alpha_A(1-\beta\alpha_B) \text{ if } p_A > p_B \end{cases} \tag{6}$$

and

$$D_B(p_A, p_B) = \begin{cases} \beta\alpha_B \text{ if } p_B < p_A \\ \beta\alpha_B\left(1 - \frac{\alpha_A}{2}\right) \text{ if } p_A = p_B. \\ \beta\alpha_B(1-\alpha_A) \text{ if } p_B > p_A \end{cases} \tag{7}$$

Comparing (1), (6) and (7), the demand functions are the same as when both consumers are shoppers and the matching probabilities for product A and product B are α_A and $\beta\alpha_B$, respectively. Therefore, the following lemma holds.

Lemma 2.

1) *Suppose that firm A wins the prominent position. Then the pricing strategy for the mixed-strategy equilibrium is the same as when all consumers are shoppers, and the matching probabilities for product A and B are α_A and $\beta\alpha_B$, respectively. The profits of the two firms are:*

$$\pi_{A,u}^{AB} = \alpha_A(1-\beta\alpha_B) \tag{8}$$

and

$$\pi_{B,u}^{AB} = \beta\alpha_B(1-\beta\alpha_B), \tag{9}$$

where $\pi_{i,u}^{ij}$ denotes the profit of firm i under UA and uniform pricing when the order of advertising position is $i \to j$.

2) *Suppose that firm B wins the prominent position. Then the pricing strategy for the mixed-strategy equilibrium is the same as when all consumers are shoppers, and the matching probabilities for product A and B are α_B and $\beta\alpha_A$, respectively. The profits of the two firms are:*

$$\pi_{A,u}^{BA} = \begin{cases}\beta\alpha_A(1-\alpha_B) \text{ if } \beta \geq \frac{\alpha_B}{\alpha_A} \\ \beta\alpha_A(1-\beta\alpha_A) \text{ if } \beta < \frac{\alpha_B}{\alpha_A}\end{cases} \tag{10}$$

and

$$\pi_{B,u}^{BA} = \begin{cases}\alpha_B(1-\alpha_B) \text{ if } \beta \geq \frac{\alpha_B}{\alpha_A} \\ \alpha_B(1-\beta\alpha_A) \text{ if } \beta < \frac{\alpha_B}{\alpha_A}\end{cases}. \tag{11}$$

The above lemma suggests that placing a product in a less prominent position is equivalent to discounting the matching probability by β. If firm A wins the prominent position, it is still the high-type firm since $\alpha_A > \beta\alpha_B$. However, if firm A loses the prominent position, the matching probability drops to $\beta\alpha_A$, which is higher than the matching probability of firm B if and only $\beta \geq \frac{\alpha_B}{\alpha_A}$. Therefore, both firms' profit functions have a kink at $\beta = \frac{\alpha_B}{\alpha_A}$.

The valuation of the prominent position for firm i can be derived as $V_{i,u} = \pi_{i,u}^{ij} - \pi_{i,u}^{ji}$, which is the difference between the profits of firm i for winning and losing the prominent position. From (8), (9), (10) and (11), we have

$$V_{A,u} = \begin{cases}(1-\beta)\alpha_A \text{ if } \beta \geq \frac{\alpha_B}{\alpha_A} \\ \alpha_A[1-\beta+\beta(\alpha_A-\alpha_B)] \text{ if } \beta < \frac{\alpha_B}{\alpha_A}\end{cases},$$

and

$$V_{B,u} = \begin{cases}\alpha_B(1-\beta)[1-(1+\beta)\alpha_B] \text{ if } \beta \geq \frac{\alpha_B}{\alpha_A} \\ \alpha_B\left[1-(1+\alpha_A)\beta+\beta^2\alpha_B\right] \text{ if } \beta < \frac{\alpha_B}{\alpha_A}\end{cases}.$$

It is widely believed that being prominent always benefits firms (Armstrong et al., 2009; Armstrong and Zhou, 2011). However, in our model, this conventional wisdom may not be true, as the following proposition states.

Proposition 1. *For high-type firm A, the prominent position is always valuable, i.e., $V_{A,u} \geq 0$ always holds. However, for low-type firm B, maintaining a prominent position may be detrimental. In particular, when $\alpha_B > 1/2$, there exists a threshold $\bar{\beta} \in (0,1)$ such that when and only when $\beta > \bar{\beta}$, the valuation of the prominent position is negative.*

For high-type firms, the benefits of prominent position are twofold. Firstly, nonshoppers only search for prominent positions, so if the firm stays in a prominent position it will get more clicks. Secondly, if the low-type firm stays in a less prominent position, price competition is mitigated due to the increasing difference in matching probability between the two firms. However, for low-type firms, there is a trade-off between exploiting a prominent position and easing the price competition. On the one hand, maintaining a prominent position allows the low-type firms to get more clicks and thus more surplus from nonshoppers. We call it the *attention-capture effect*, which adds value to the prominent position. On the other hand, low-type firms now face fiercer price competition from high-type firms who are eager to capture shoppers by lowering their prices, creating a*price-competition effect*, which reduces the value of the prominent position. As the proportion of shoppers increases, low-type firms become more concerned with price competition. In particular, when $\beta > \bar{\beta}$, the *price-competition effect* dominates the *attention-capture effect* and, the value of the prominent position therefore becomes negative. In this case, low-type firms prefer to pay in order to avoid being placed in a prominent position.

In a related study, Xu et al. (2011) showed that prominent positions may have no value if the price of a product market is endogenous. Contrary to our results, strong firms may not value prominent positions, while weak firms always value them. This is because, in their model, firms have the same matching probability but different marginal production costs, whereas in our model, firms have different matching probabilities but identical production costs.

It is more in line with common sense that the prominent position is of value to both firms, except in the case of $\alpha_B > 1/2$ and $\beta > \bar{\beta}$. Therefore, our subsequent analysis will focus on these cases. That is, we assume that either $\alpha_B \le 1/2$ or $\alpha_B > 1/2$ and $\beta \le \bar{\beta}$.

In the second-price auction, both firms bid truthfully for the prominent position. Thus, firm A bids $V_{A,u}$ and firm B bids $V_{B,u}$. if $V_{i,u} \ge V_{j,u}$, firm i wins the position auction and pays $V_{j,u}$ to the publisher.

Lemma 3. *High-type firm A always outbids low-type firm B, and the publisher receives a payoff of $V_{B,u}$ from the position auction.*

The high-type firm always has a greater incentive to be placed prominently and thus wins the position auction. Firstly, prominence means more clicks from nonshoppers. High-type firm has a higher matching probability and therefore benefits more from clicks from nonshoppers than the low-type firm. In addition, prominence generates a higher value to the high-type firm because price competition is mitigated (intensified) when the high-type firm (the low-type firm) is in the prominent position.

4.2. Targeted advertising based on product preference

Suppose the publisher targets advertising based on consumers' product preferences (TP). The overall market is therefore divided into two submarkets, each consisting of consumers with the same product preferences. Suppose submarket i consists of all type i consumers, where $i = A, B$.

In submarket i, the matching probability is α_h for firm i, and α_l for firm $j \ne i$, for each $i = A, B$. Clearly, in this subsection, the auction game and the subsequent pricing game are just special cases of UA with $\gamma = 1$ in the previous subsection. Thus, we have the following results:

Lemma 4. *Suppose the publisher implements TP.*

i) *Firm i always wins the prominent position in submarket i, $i = A, B$;*
ii) *In submarket i, firms' prices and profits are given by (2)–(5), replacing α_1 by α_h and α_2 by $\beta\alpha_l$. In particular, the profits are given by:*

$$\pi^i_{i,tp} = \alpha_h(1 - \beta\alpha_l)$$

and

$$\pi^i_{j,tp} = \beta\alpha_l(1 - \beta\alpha_l),$$

where $\pi^i_{j,tp}$ represents the expected profit of firm j in submarket i under TP.
iii) *The total auction payoff to the publisher from the position auction is given by:*

$$V_{tp} = \begin{cases} \alpha_l(1-\beta)[1-(1+\beta)\alpha_l] \text{ if } \beta \ge \frac{\alpha_l}{\alpha_h} \\ \alpha_l\left[1-(1+\alpha_h)\beta + \beta^2\alpha_l\right] \text{ if } \beta < \frac{\alpha_l}{\alpha_h} \end{cases}.$$

4.3. Targeted advertising based on search preference

Suppose a publisher has implemented targeted advertising based on consumer search preferences (TS). The whole market can be divided into two submarkets, one for the shoppers and the other for the nonshoppers. For the submarket of shoppers, since shoppers always search for both firms, the prominent position has no value to the firms. As a result, the online publisher obtains a zero payoff from the position auction. In the product market, firms' pricing strategies are given in Lemma 1 with $\alpha_1 = \alpha_A$ and $\alpha_2 = \alpha_B$.

Now we consider the submarket of nonshoppers. Nonshoppers seek only the advertised product, as long as the product matches their preferences and the price is no higher than 1. The prominent firm will thus charge a monopoly price of 1. And the firm in the second position receives no clicks and therefore receives zero profit. Therefore, in this submarket, the valuation of the prominent position is α_A for firm A, and α_B for firm B. Since $\alpha_A \ge \alpha_B$, firm A always wins the position auction and pays α_B to the publisher. Under TS, the publisher's total expected payoff from the position auction is equal to $(1-\beta)\alpha_B$.

4.4. Targeted advertising based on both preferences

Suppose that the publisher has implemented targeted advertising based on both consumer search and product preferences (T2). Similar to TS, the prominent positions in the shopper submarkets generate no value to the firms, and therefore the position auctions in these submarkets result in zero revenue. For each $i = A, B$, the pricing equilibrium in the submarket with type i shoppers is given by Lemma 1 with $\alpha_1 = \alpha_h$ and $\alpha_2 = \alpha_l$.

For nonshopper-submarkets, the prominent firm always charges the nonshopper a monopoly price of 1, while the non-prominent firm receives no clicks and therefore earns zero profit. In the submarket with type i nonshoppers, the prominent position is valued as α_h for firm i and α_l for firm $j \ne i$, where $i, j \in \{A, B\}$. Therefore, each firm wins the position auction in its strong submarket and pays α_l to the publisher. The total expected revenue from the position auctions is thus $(1-\beta)\alpha_l$.

5. Comparison

This section compares auction revenues, equilibrium prices, industry profits and consumer surplus for different targeting techniques.

5.1. Comparison between UA and TP

We first compare firms' pricing strategies in the product markets between TP and UA. For any $i, j \in \{A, B\}$, let $\tilde{p}_{i,u}$ denote firm i's stochastic pricing strategy under UA, and $\tilde{p}^i_{j,tp}$ denote firm j's stochastic pricing strategy in the submarket with type i consumers under TP.

Lemma 5. *The pricing strategy $\tilde{p}^i_{i,tp}$ first-order stochastically dominates $\tilde{p}_{A,u}$ and $\tilde{p}^i_{j,tp}$ first-order stochastically dominates $\tilde{p}_{B,u}$, for $i, j \in \{A, B\}$ and $j \neq i$.*

The above lemma shows that equilibrium prices under TP are higher than those under UA. Note that the high-type firm always wins the position auctions. The matching probabilities for the two firms are α_A and α_B under UA, and α_h and α_l under targeted advertising, respectively. Recall that $\alpha_A = \gamma\alpha_h + (1-\gamma)\alpha_l \leq \alpha_h$ and $\alpha_B = \gamma\alpha_l + (1-\gamma)\alpha_h \geq \alpha_l$, i.e., the difference in matching probabilities between firms is greater under TP. The stronger product differentiation mitigates price competition under TP, so that both firms charge higher prices in the sense of first-order stochastic dominance.

The following proposition compares the publisher's auction payoffs between TP and UA.

Proposition 2. *There exists a threshold value $\hat{\gamma}_0 \in [1/2, 1]$, so that (i) the publisher's auction payoff is higher under UA than under TP whenever $\gamma \geq \hat{\gamma}_0$; (ii) when $\gamma < \hat{\gamma}_0$, there exists a threshold value $\hat{\beta} \in (\frac{\alpha_l}{\alpha_h}, 1]$, so that the publisher's auction payoff is higher under TP than under UA if and only if $\beta > \hat{\beta}$.*

From Lemma 3, the publisher's auction payoff equals the low-type firm's valuation of prominence. The low-type firm trades off between the *attention-capture effect* and the *price-competition effect*. Due to the *attention-capture effect*, the low-type firm is willing to pay more for the prominent position under UA than under TP, as the low-type firm has a higher matching probability under UA and can retain nonshoppers better after being prominent. Due to the *price-competition effect*, the low-type firm is willing to pay less for the prominent position under UA than under TP. This is because, under UA, the low-type firm has a greater incentive to avoid intense price competition by not being prominent.

If γ is large enough such that the difference in matching probabilities between the two firms is very large, then price competition under UA is sufficiently moderated and the incentive to soften price competition is sufficiently low. In this case, the *attention-capture effect* dominates the *price-competition effect*, implying that UA is superior to TP. However, if γ is small, i.e., close to $1/2$, the difference in the matching probabilities between the two firms is small. This makes the low-type firm more concerned with the *price-competition effect*, as it becomes more important to mitigate price competition. Whether the *attention-capture effect* remains dominant in this case depends on the proportion of nonshoppers. Specifically, the *attention-capture effect* dominates the *price-competition effect* if and only if the proportion of nonshoppers is sufficiently high, i.e., if β is small.

In Chen and Stallaert's (2014) study, the publisher always tend to choose UA when there are only two firms competing for a prominent position, which contrasts with our results. The reason for this is that in their model the firms' pricing strategies are exogenous, while in our model they are endogenous. In the case of exogenous product prices, TP only mitigates firms' competition in the position auction, leading to a lower payoff for the publisher. However, when product prices are endogenous, TP also mitigates firms' price competition in the subsequent product market, thereby increasing firms' valuations of the prominent position.

Next, we look at firms' profits and industry profits.

Proposition 3.

1) *Industry profits are higher under TP than under UA.*
2) *Firm B always prefers TP to UA, while firm A always prefers TP to UA at small and large values of β, but may prefer UA to TP at intermediate values of β.*

The intuition for result 1) is straightforward. Firstly, TP mitigates price competition in each submarket compared to UA. Secondly, TP allocates the prominent position to the strong firm in each submarket so that consumers are always matched the products they liked best, i.e., type i product always appears prominently to type i consumers, for each $i = A, B$. As a result, firms are more likely to retain nonshoppers under TP, which generates higher profits.

For result 2), the low-type firm B always loses position auction under UA, but wins prominence in its strong submarket (i.e., submarket with type B consumers) under TP. Furthermore, price competition in the product market is mitigated under TP. Therefore, firm B always tends to prefer TP. However, for the high-type firm A, TP has two opposite effects. On the one hand, compared to UA, TP eases competition in the product market and thus generates higher profits. On the other hand, firm A suffers a loss of profit under TP because it loses its prominence in a weak submarket, whereas under UA it always appears to be prominent.

Finally, we discuss the impact of TP on consumer surplus and total welfare.

Proposition 4.

1) *There exists a threshold value $\hat{\gamma}_1 \in (1/2, 1)$ such that type B nonshoppers prefer TP if and only if $\gamma < \hat{\gamma}_1$. All other types of consumers (type A nonshoppers, type A shoppers, and type B shoppers) prefer UA. Total consumer surplus is higher under UA than under TP.*
2) *Total surplus is higher under TP than under UA.*

Shoppers always search for both firms. As a result, they are only concerned with the expected prices. Targeted advertising eases price competition for firms in the product market, so the expected prices are higher, which ultimately hurts the shoppers. For type *A* nonshoppers, product *A* appears prominently to them under both UA and TP. Therefore, they only care about price and prefer UA. For type *B* nonshoppers, product *A* appears prominently under UA and product *B* appears prominently under TP. In other words, for type *B* nonshoppers there is a trade-off: the product that appear prominently to them has a higher matching probability but also a higher price under TP than under UA. As a result, they may or may not prefer TP. As TP only benefits type *B* nonshoppers to the detriment of all other types of consumers, its overall impact on total consumer surplus is negative.

The total surplus depends only on the probability of trading. Shoppers always search for both firms and will buy as long as at least one of them matches their preferences; therefore, TP does not affect shoppers' trading probabilities. On the other hand, nonshoppers only search for the prominent products and therefore, under TP, they always visit their favorites products in each submarket first. Thus, TP increases the trading probability and therefore the total surplus compared to UA,.

5.2. Comparison among UA, TS, TP and T2

Advances in information technology have enabled online publishers to collect detailed information about online consumer activity, on the basis of which online publishers can infer not only consumers' product preferences, but also their search preferences. As a result, publishers can implement targeted advertising based on full or partial information about preference types. The following proposition suggests that targeted advertising based on both types of preferences (T2) is never optimal for publishers.

Proposition 5. *TS always increases the publisher's auction payoff. Conditional on TS, TP decreases the publisher's auction payoff. Thus, of the four possible advertising techniques: UA, TP, TS, T2, the publisher receives the highest payoff from the position auction under TS.*

The publisher's payoff is equal to the valuation of the prominent position by the low-type firm. The prominent position is valuable to the firms because nonshoppers only visit and buy from the prominent firm. However, if the publisher implements TS, the firm in the prominent position can charge nonshoppers a monopoly price. Therefore, implementing TS always increases the publisher's payoff.

Note that T2 can be considered as a combination of TS and TP. Thus, we can compare TS and T2 by asking the question that whether the publisher should additionally conduct TP when TS has already been implemented. Conditional on TS, the publisher's payoff is equal to the monopoly profit that the low-type firm receives from nonshoppers, which increases with the matching probability for the low-type firm. If the publisher additionally implements TP, then the matching probability for low-type firms decreases from α_B to α_l in each submarket with nonshoppers. Thus, TP reduces the monopoly profit of the low-type firm and thus reduces the publisher's auction payoff. Our results suggest that the two types of targeting techniques, TS and TP, are substitutes for each other. The publisher may prefer to implement TP if TS is not feasible. As long as TS is feasible, the publisher will not implement TP.

We now proceed to the welfare analysis. The following proposition shows how TS affects welfare.

Proposition 6.

1) *TS hurts both firms A and B and reduces total industry profits.*
2) *TS hurts nonshoppers, benefits shoppers, and increases total consumer surplus.*
3) *TS has no effect on total surplus.*

The above proposition implies that the impact of TS on welfare does not depend on whether the publisher has implemented TP. In other words, the proposition holds when comparing UA and TS or TP and T2. For example, result 1) implies that both firms prefer UA to TS and TP to T2.

TS hurts both firms. In the submarket with shoppers, TS increases price competition and therefore lowers firms' profits. For the nonshopper submarkets, the low-type firm always loses in the position auction and earns zero profit, while the high-type firm makes a higher profit in the product market but also pays more to the publisher in the position auction. The overall effect of TS on each firm's net profit is negative.

The result for consumer surplus is straightforward. TS increases price competition for firms facing shoppers, but eases competition facing nonshoppers. Thus, TS is benefitial to shoppers and detrimental to nonshoppers. The gains for shoppers outweigh the losses for nonshoppers and, therefore, total consumer surplus increases.

The total surplus depends only on the trading probability. The implementation of TS only changes equilibrium prices, not the advertisements displayed prominently to each consumer. Therefore, it has no effect on the trading probability and total surplus.

The table below summarizes our analysis.

6. Targeted advertising without price discrimination

This section examines the situation where price discrimination is not permitted in product markets. Here, targeted advertising is still permitted, and firms can bid for prominent positions in each submarket. The following proposition suggests that once price discrimination is prohibited, information about consumers' search preferences is meaningless.

Proposition 7. *Assuming that price discrimination is prohibited, then TS has no effect on a firm's pricing strategy in the product market. In other words, from the perspective of firms' pricing strategies, UA is equivalent to TS and TP is equivalent to T2.*

Intuitively, the advantage of distinguishing between nonshoppers and shoppers is that firms can charge higher prices to nonshoppers, as nonshoppers are less likely to compare prices. However, when firms are forced to charge the same price to shoppers and nonshoppers, the distinction between shoppers and nonshoppers becomes meaningless.

Since firms' valuation of prominence is determined by price competition in the product market, the above proposition suggests that TS has no effect on firms' bidding behavior and hence on publishers' auction payoffs. Therefore, we only need to consider the case of UA and TP, since TS is equivalent to UA and T2 is equivalent to TP.

When price discrimination is prohibited, the TP situation under uniform pricing is complex. Since prices must be the same in all submarkets, a firm's valuation of a prominent position in one submarket depends on whether the firm wins prominent positions in other submarkets. This interdependence of valuations makes it difficult to analyze position auctions when prominent positions are auctioned simultaneously. To simplify our analysis, we consider only the case of symmetric firms, i.e., $\gamma = 1/2$, and assume that prominent positions are auctioned sequentially. Without loss of generality, we assume that the publisher first auctions off the prominent position in the type A submarket, and then in the type B submarket.

We first examine price competition in the product market. The UA case, already analyzed in Section 3.1, is equivalent to the case where all consumers are shoppers and the matching probability of the firm in a prominent position is $\frac{\alpha_h+\alpha_l}{2}$ and the matching probability of the other firm is $\frac{\beta(\alpha_h+\alpha_l)}{2}$. In equilibrium, the profit of the prominent firm is $\pi_u^h = \frac{\alpha_h+\alpha_l}{2}\left(1 - \frac{\beta(\alpha_h+\alpha_l)}{2}\right)$ and of the other is $\pi_u^l = \beta\pi_u^h$. The industry profit is $\Pi_u = \pi_u^h + \pi_u^l = (1+\beta)\pi_u^h$. In the position auction, each firm bids $\pi_u^h - \pi_u^l$ has a winning probability of 1/2. The publisher's auction payoff is equal to $\pi_u^h - \pi_u^l$.

Under TP, there are four possible scenarios for the final winning position. 1) A wins the type A submarket and B wins the type B submarket; 2) A wins the type B submarket and B wins the type A submarket; 3) A wins both submarkets; 4) B wins both submarkets.

Case 1) is equivalent to a model in which all consumers are shoppers and both firms have matching probability $\frac{\alpha_h+\beta\alpha_l}{2}$. The revenue of each firm is $\pi_1 = \frac{\alpha_h+\beta\alpha_l}{2}\left(1 - \frac{\alpha_h+\beta\alpha_l}{2}\right)$ and the industry profit is $\Pi_1 = 2\pi_1$. Case 2) is equivalent to the model in which all consumers are shoppers and both firms have matching probability $\frac{\alpha_l+\beta\alpha_h}{2}$. Each firm earns $\pi_2 = \frac{\alpha_l+\beta\alpha_h}{2}\left(1 - \frac{\alpha_l+\beta\alpha_h}{2}\right)$ and the industry profit is $\Pi_2 = 2\pi_2$. Case 3) and 4) are equivalent to UA.

We now solve the sequential auction under TP by backward induction.

Lemma 6.

1) *Conditional on firm A winning the type A submarket, either firm A or firm B can win the type B submarket.*
2) *Conditional on firm B winning the type A submarket, firm B always wins the type B submarket.*

As firms must charge a uniform price, the valuation of prominent positions in the two submarkets is interdependent. Conditional on winning the first auction, firms have a greater incentive to win subsequent auctions. Thus, if a firm wins its strong submarket in the first auction, it is likely to win its weaker submarket in the second auction as well. This result stands in stark contrast to the case of price discrimination, where firms never win their weaker submarkets.

Now, consider the first stage of the auction. The following lemma shows the outcome for winners in each submarket.

Lemma 7. *Suppose the publisher implements TP. If $\Pi_u \geq \Pi_1$, then either firm A or firm B wins both submarkets. If $\Pi_u \leq \Pi_1$, then firm A wins the type A submarket and firm B wins the type B submarket.*

Finally, we examine whether publishers should implement TP.

Proposition 8. *Assume that price discrimination is prohibited, the publisher's auction payoff is lower under TP than under UA.*

Since the two auctions are interdependent, there are cases where a firm can win both submarkets even if the publisher implements TP. In this case, the advertising and pricing results for the product market are the same as under UA. However, the publisher's auction payoff is lower than under UA. Intuitively, under TP, if firm i wins the first auction, it will win both submarkets. If it loses the first auction, it will lose both submarkets. The difference between the profit of winning and losing

Table 1
The most preferred advertising technique from different perspectives.

Publisher	Firm A	Firm B
TS	TP/UA	TP
Shoppers	Type-A nonshoppers	Type-B nonshoppers
TS	UA	TP/UA
Industry profit	Consumer surplus	Total welfare
TP	TS	TP and $T2$

the first auction is equal to the valuation of the prominent position under UA, which is the publisher's auction payoff under UA. Firm i 's valuation of winning the first auction is equal to the profit difference above minus firm i 's future payment for winning the second auction. If firm i pays its own valuation for the first auction, then TP generates the same auction payoff for the publisher as UA. However, as the auction is a second-price auction, firm i pays less than its own valuation in the first auction, which means that the total auction payoff under TP should be lower than under UA.

There are also situations where each firm wins its strong submarket, leading to the same advertising results as under TP with price discrimination. However, the implementation of TP is no longer an advantage for the publisher, which contrasts with the case of price discrimination. As mentioned earlier, the advantage of implementing TP is that firms are relieved of price competition in their product markets, as each firm has a high degree of monopoly power in its strong submarket. However, when price discrimination is prohibited, this advantage is limited because firms must take the entire market into account when pricing. As a result, the disadvantage of implementing TP prevails due to weaker competition in position auctions, which leads to lower auction payoffs under TP.

In summary, our results suggest that publishers have no incentive to implement targeted advertising in the absence of price discrimination. Chen and Stallaert (2014) also show that publishers always choose not to implement targeted advertising if the valuation of the firm is exogenous and there are only two firms. Thus, our results generalise their results by enabling endogenous prices in the product market and including targeted advertising based on search preferences. These results appear to be invalid, as we see targeted advertising frequently in reality. Note, however, that we only studied two firms in this article. As suggested by Chen and Stallaert (2014), targeted advertising would benefit publishers if there were more than two firms. In addition, the targeted advertising we studied involved segmenting the consumer market based on demand, whereas in reality, targeted advertising also involves the precise delivery of advertising messages to otherwise uninformed consumers. While it is always beneficial to provide accurate advertising messages to uninformed consumers, our results suggest that segmenting consumer markets may be detrimental to publishers. The message that this article attempts to convey is that one of the advantages of TP is that it mitigates price competition, which is maximized while allowing for price discrimination.

We now discuss the welfare implications of prohibiting price discrimination. In targeted advertising with price discrimination, the publisher chooses TS in equilibrium. In targeted advertising without price discrimination, the equilibrium is UA. Therefore, we only need to compare TS and UA. From proposition 6, we know that TS has no effect on total welfare, always increasing total consumer surplus and decreasing industry profit. Thus, we have the following table.

7. Policy implications

This section discusses the policy implications in light of the previous analysis. For the case where price discrimination is permitted, we investigate policy interventions under different primary goals of competition law. We then discuss what happens and the scope of different advertising techniques if consumers have control over their personal data.[10] Finally, we discuss the policy implications regarding price discrimination in light of the analysis in Section 6.

Policy interventions depend on the overall mission of competition policy in a given jurisdiction, i.e., whether consumer welfare or total welfare is the primary objective. As mentioned by Motta (2004), in the US, both courts and antitrust agencies seem to favor the consumer welfare objective, while in other jurisdictions, such as Canada, New Zealand and Australia, competition authorities seem to favour the general welfare criterion. In what follows, we will consider both objectives and discuss the policy interventions in each case.

We begin with a discussion of the cases where price discrimination is permitted. Table 1 shows that total welfare is the same and highest under TP and T2. However, in equilibrium, the publisher will choose TS. Thus, when the publisher has information about consumers' search preferences, it will make too little use of information about consumers' product preferences. Policymakers concerned about total welfare should adopt policies that encourage publishers to collect and use data on consumers' product preferences, but discourage publishers from collecting and using data on consumers' search preferences.

Table 1 also shows that consumer surplus is highest under TS and that the publisher also chooses TS in equilibrium. The point 71 (on page 29) of the OECD (2018) report on Personalized Pricing establishes that the consumer welfare criterion is used by most jurisdictions: *"According to a survey by the ICN, in 89% of the jurisdictions consumer welfare is the primary goal*

[10] We thank an anonymous referee for the suggestions on policy implications in this section.

Table 2
The effect on welfare of the prohibition of price discrimination.

Industry profit	Consumer surplus	Total welfare
$-$	$+$	unchanged

or one of the goals of competition law (Figure 13)... " Hence, our analysis shows that there is no reason for policy interventions in most jurisdictions.

Our model can also shed light on how consumers can provide or hide their personal data. Data privacy regulations in the EU (General Data Protection Regulation)[11] and China[12] allow online consumers to completely remove all tags, including those passively tagged by artificial intelligence algorithms and those voluntarily provided by consumers. Empowered by these regulatory policies, consumers are free to control their personal data. From Proposition 4 and Proposition 6 (see also Table 1), shoppers are better off under TS, but worse off under TP. Thus, shoppers always have incentives to disclose their search preferences, but hide their product preferences. On the other hand, nonshoppers always tend to hide their search preferences because they are worse off under TS. It is worth noting that even though consumers have control over personal data, online publishers can still infer the search preferences of all consumers: all shoppers disclose their search preferences, while those who do not disclose their search preferences are nonshoppers. However, shoppers' product preferences can no longer be inferred. Finally, according to Proposition 4, whether a nonshopper is motivated to reveal her product preferences depends on her type: nonshoppers loyal to large brands tend to hide their product preferences, while nonshoppers loyal to small brands may reveal their product preferences if and only if the brand differences between the products are not too great. In other words, nonshoppers' product preferences can only be inferred by online publishers if the two brands are close in popularity.

According to Table 2, the policy implications regarding price discrimination are clear. A policy maker who is only concerned with total welfare is indifferent to implementing or prohibiting price discrimination. If the policymaker also cares about consumer surplus, then she will be more willing to prohibit price discrimination. In the absence of price discrimination, consumer control over personal data becomes irrelevant because in this case, online publishers have no incentive to implement targeted advertising.

8. Extensions

Commission fees. We have assumed that the online publisher only generates revenue from their advertising marketplace. This applies to search engines (e.g., Google and Baidu) and social media (e.g., Facebook and Tencent). However, e-commerce platforms (e.g., Amazon and Alibaba's Taobao Mall) also profit from commission fees from sellers in their product marketplaces. For example, Taobao Mall charges a 2% - 5% fee and Amazon charges 10% of the final price in most categories.

The revenue composition of a publisher can influence the optimal choice of targeting technique. From Corollary 1, if the publisher's main revenue comes from position auctions, then TS appears to be the best choice. However, if the main revenue comes from commission fees charged in the product marketplace, then the firm should choose a target technique that maximises industry profits, i.e. TP. Our results suggest that different online platforms may be interested in different dimensions of consumer data. E-commerce platforms are more interested in data on consumers' product preferences, while search engines and social media are more likely to collect data on consumers' search preferences.

Endogenous targeting quality. In the previous analysis, we have assumed that the publisher chooses between perfect targeting and no targeting, and showed that both can be optimal, depending on the parameters. This suggests that if we consider continuous targeting quality, publishers' auction revenue may increase or decrease with targeting quality. One wonders, then, whether it is optimal for the publisher to choose a modest level of targeting quality. We show that for the symmetric case defined below, it is always optimal for the publisher to choose extreme targeting quality. Therefore, it is harmless to consider only perfect targeting and UA.

Assuming that the publisher does not observe true preferences, but rather noisy signals of consumer preferences, the publisher is free to choose the targeting quality defined below.

Suppose that the publisher implements TP. For simplicity, we consider the case of symmetric firms, where $\gamma = 1/2$. The publisher observes a noisy signal σ_i based on the type of each consumer, with $\Pr(\sigma_i|\text{type-}i) = a \geq 1/2$, for each $i = A, B$. The probability a can thus be interpreted as the quality of targeting, which can be optimally chosen by the publisher. The cases of UA and perfect targeting correspond to $a = \frac{1}{2}$ and $a = 1$, respectively.

Due to symmetry, the probability of observing σ_i is equal to $1/2$. Using Bayes' rule, we can easily obtain that $\Pr(\text{type-}i|\sigma_i) = a$. Thus, conditional on σ_i, the matching probability for product i is $\alpha^i_{\sigma_i} = a\alpha_h + (1-a)\alpha_l$ and the matching probability for product j is $\alpha^j_{\sigma_i} = a\alpha_l + (1-a)\alpha_h$. Note that $\alpha^i_{\sigma_i} > \alpha^j_{\sigma_i}$; therefore, firm i is the strong firm which wins prominence in the type σ_i submarket.

[11] See Articles 7, 16, and 17 of GDPR: https://gdpr-info.eu/.

[12] See Articles 13, 16 and 47 of Personal Information Protection Law of the People's Republic of China: http://www.npc.gov.cn/npc/c30834/202108/a8c4e3672c74491a80b53a172bb753fe.shtml.

The publisher's auction payoff is given by:

$$V = \begin{cases} \alpha^j_{\sigma_i}(1-\beta)\left[1-(1+\beta)\alpha^j_{\sigma_i}\right] \text{ if } \beta \geq \frac{\alpha^j_{\sigma_i}}{\alpha^i_{\sigma_i}} \\ \alpha^j_{\sigma_i}\left[1-\beta-\alpha^i_{\sigma_i}\beta+\beta^2\alpha^j_{\sigma_i}\right] \text{ if } \beta \leq \frac{\alpha^j_{\sigma_i}}{\alpha^i_{\sigma_i}} \end{cases}.$$

Proposition 9. *Given any $0 < \alpha_l < \alpha_h < 1$ and $0 < \beta < 1$, the auction payoff V always reaches its maximum at $a = 1/2$ or $a = 1$. That is, the publisher only chooses perfect targeting or no targeting.*

Bergemann and Bonatti (2011) investigated how improved targeting techniques affect ad prices in the advertising market. They showed that an increase in targeting quality rotates the demand curve for advertising and therefore first increases and then decreases the equilibrium price of advertising. At moderate targeting quality, the equilibrium price of advertising reached a maximum. Here, we study a similar problem in the context of online advertising, namely position auctions, and show that publishers' revenues are maximised at the extremes of targeting quality. Both the setup and the results differ from theirs. Gal-Or et al. (2006) investigated how advertisers should allocate resources to improve targeting quality. They distinguished between two types of targeting quality: accuracy and recognition. Accuracy was defined as the probability that a consumer is actually a type i consumer, provided that he/she is observed to be a type i consumer from the data (i.e. $\Pr(\text{type-}i|\sigma_i)$), while recognition was defined as the probability of being identified as a type i consumer, as long as the consumer is a type i consumer. The authors showed that an increase in accuracy does not affect price competition, while an increase in recognition increases price competition and reduces industry profit. In our model, these two parameters degenerate to a single parameter a, and an increase in a may either increase or decrease industry profits.

Now suppose the publisher implements TS. The publisher observes σ_i, $i = s, n$, where s represents shopper and n represents nonshopper. For each i, suppose that $\Pr(\sigma_i|\text{type-}i) = b$, with $b \geq 1/2$. To simplify our analysis, we consider the case $\beta = \frac{1}{2}$. The probability of observing σ_i is $\frac{1}{2}$. Using Bayes' rule, we have $\Pr(\text{type-}i|\sigma_i) = b$. In both the σ_s market and the σ_n market, firm A is the winner.

Proposition 10. *Suppose that the publisher implements TS and $\beta = 1/2$. Then the publisher's auction payoff is increasing in the targeting quality b.*

With imperfect targeting, the prominent positions for both types of consumers are valuable to firms. Moreover, when firms observe σ_n, they are unable to charge monopoly prices. An increase in the targeting quality b eases price competition in the σ_n market, but intensifies price competition in the σ_s market. As a result, with the increase b, publishers increase their auction revenue in the σ_n market and decrease their revenue in the σ_s market. The above proposition suggests that increasing the targeting quality of TS will have an overall positive effect on the publisher's revenue.

9. Conclusions

In this article, we examine the economic impact of different targeting techniques. We show that targeted advertising based on consumer product preference (TP) has a fundamentally different impact from targeted advertising based on consumer search preference (TS). The implementation of TP mitigates price competition among firms in product markets, which benefits the publisher, but also mitigates competition among firms in position auctions, which detriments the publisher. As a result, publishers may or may not benefit from the implementation of TP. However, the implementation of TS mitigates price competition by allowing firms to charge monopoly prices to nonshoppers, but intensifies price competition among firms facing shoppers. Publishers always benefit from the implementation of TS. If a publisher is free to choose between TP and TS, it will make the best decision to implement only TS.

TP improves the match between consumers and products, thereby increasing social welfare. However, it also reduces price competition between firms in the product market and increases industry profit at the expense of consumer surplus. TS, on the other hand, does not change the probability of matching between consumers and products, and therefore has no effect on social welfare. It does, however, lead to higher prices for nonshoppers and lower prices for shoppers. The overall effect of TS on consumer surplus is positive, while the effect on industry profit is negative. These welfare analyses have important policy implications for whether to impose strict data protection rules on the use of private consumer data.

The present research can be extended along several lines. First, the model could be extended to more than two firms. As Chen and Stallaert (2014) pointed out, the number of firms may be an important factor in determine a publisher's choice of targeted advertising. The difficulty is that once there are more than two firms, there are multiple equilibria in the pricing game of the product market. In addition, analyzing position auctions becomes more complex as firms must bid for multiple positions. Second, we assumed that consumers are either shoppers who always search for both firms or nonshoppers who only search for the firm recommended by advertisement. However, it is more realistic to assume that consumers have heterogeneous but positive search costs and make an endogenous decision about whether to search for another firm. In our current model, shoppers correspond to consumers with low search costs, while nonshoppers correspond to consumers with high search costs. Finally, consumers may be harmed by targeted advertising and may therefore take actions to avoid being tracked by online publishers, such as deleting and blocking browser cookies, using multiple online identities, or strategically buying from unrelated sellers. It would therefore be interesting to include endogenous consumer decisions in our model and to examine the strategic interactions between consumers, firms and online publishers.

CRediT authorship contribution statement

Sanxi Li: Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing. **Hailin Sun:** Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing. **Jun Yu:** Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing.

Data availability

No data was used for the research described in the article.

In this section, we provide technical proofs for all the lemmas and propositions in the main text.

Proof of Lemma 1. The model is essentially equivalent to Narasimhan (1988) in that for $i, j \in \{1, 2\}$ and $i \neq j$, there is $\alpha_i\left(1-\alpha_j\right)$ proportion of consumers loyal to firm i and there is a faction of $\alpha_i\alpha_j$ consumers who are switching consumers and buy from the lower priced firm. According to Proposition 1–5 of Narasimhan (1988), the equilibrium pricing strategy must be mixed for both firms, with the following properties: (i) the price support of each firm is convex (i.e. there is no gap in price support); (ii) the upper and lower bounds on price support are the same for both firms, with the upper bound equal to 1; and (iii) the strong firm, i.e. firm 1, has at the upper bound a mass point, which is the unique mass point in the price distribution for both firms.

Let $F_i(p)$ be the distribution function of firm i's mixed strategy with $i = 1, 2$. Let p_0 be the common lower bound on price support. Let $m \in (0, 1)$ be the measure of mass point of firm 1 at $p = 1$, i.e. $F_1(1) = 1 - m$.

To solve the equilibrium problem, note that the expected profit of firm 1's is given by

$$\pi_1 = p_0\alpha_1 = \alpha_1(1-\alpha_2), \tag{12}$$

where the first equation holds when firm 1's price is equal to the lower bound p_0, and the second equation holds when its price equals the upper bound $p = 1$.

The expected profit of firm 2 is given by

$$\pi_2 = p_0\alpha_2 = (1-m)\alpha_2(1-\alpha_1) + m\alpha_2, \tag{13}$$

where the first equation holds when firm 2's price is equal to the lower bound, and the second holds when its price approaches the upper bound $p = 1$ from below, so that with probability $(1 - m)$, firm 2 attracts only its loyal consumers, and with probability m, firm 2 attracts both loyal and switching consumers.

It follows from (12) and (13) that

$$\begin{aligned} p_0 &= (1-\alpha_2) \\ m &= 1 - \frac{\alpha_2}{\alpha_1} \\ \pi_1 &= \alpha_1(1-\alpha_2) \\ \pi_2 &= \alpha_2(1-\alpha_2) \end{aligned}$$

Finally, the distribution function $F_i(p)$ can be easily derived from the following constant payoff condition

$$\pi_i = p \cdot \left[\alpha_i\left(1-\alpha_j\right) + \alpha_i\alpha_j\left(1-F_j(p)\right)\right] \text{ for any } p \in [(1-\alpha_2), 1)$$

with $i, j \in \{1, 2\}$ and $i \neq j$. □

Proof of Lemma 2. Recall that, under UA, the demands of the two firms are:

$$D_A(p_A, p_B) = \begin{cases} \alpha_A \text{ if } p_A < p_B \\ \alpha_A\left(1 - \frac{\beta\alpha_B}{2}\right) \text{ if } p_A = p_B \\ \alpha_A(1-\beta\alpha_B) \text{ if } p_A > p_B \end{cases} \tag{14}$$

and

$$D_B(p_A, p_B) = \begin{cases} \beta\alpha_B \text{ if } p_B < p_A \\ \beta\alpha_B\left(1 - \frac{\alpha_A}{2}\right) \text{ if } p_A = p_B \, . \\ \beta\alpha_B(1-\alpha_A) \text{ if } p_B > p_A \end{cases} \tag{15}$$

On the other hand, if all consumers are shoppers, the demand of firm i is

$$D_i\left(p_i, p_j\right) = \begin{cases} \alpha_i \text{ if } p_i < p_j \\ \alpha_i\left(1 - \frac{\alpha_j}{2}\right) \text{ if } p_i = p_j \, . \\ \alpha_i\left(1-\alpha_j\right) \text{ if } p_i > p_j \end{cases} \tag{16}$$

Comparing (14), (15) and (16), the demand functions are the same as when both firms are shoppers, and the matching probabilities of product A and product B are α_A and $\beta\alpha_B$ respectively. Therefore, Lemma 2 follows immediately from the results of Lemma 1. □

Proof of Proposition 1. Recall that under UA, the valuation of each firm's prominent position is as follows.

$$V_{A,u} = \begin{cases} (1-\beta)\alpha_A \text{ if } \beta \geq \frac{\alpha_B}{\alpha_A} \\ \alpha_A[1-\beta+\beta(\alpha_A-\alpha_B)] \text{ if } \beta < \frac{\alpha_B}{\alpha_A} \end{cases}, \tag{17}$$

and

$$V_{B,u} = \begin{cases} \alpha_B(1-\beta)[1-(1+\beta)\alpha_B] \text{ if } \beta \geq \frac{\alpha_B}{\alpha_A} \\ \alpha_B\left[1-(1+\alpha_A)\beta+\beta^2\alpha_B\right] \text{ if } \beta < \frac{\alpha_B}{\alpha_A} \end{cases}. \tag{18}$$

$V_{A,u} \geq 0$ always holds from (17). From (18), since $\alpha_A \geq \alpha_B$, the sign of $V_{B,u}$ is the same as that of $\tilde{V}_{B,u}$, where

$$\tilde{V}_{B,u} = \begin{cases} \left(1-\frac{\alpha_B}{\alpha_A}\right)[1-(1+\beta)\alpha_B] \text{ if } \beta \geq \frac{\alpha_B}{\alpha_A} \\ 1-(1+\alpha_A)\beta+\beta^2\alpha_B \text{ if } \beta < \frac{\alpha_B}{\alpha_A} \end{cases}.$$

We can easily verify that $\tilde{V}_{B,u}$ is decreasing and continuous in β for any $\beta \in (0,1)$. Note that the maximum value of $\tilde{V}_{B,u}$, i.e. when $\beta = 0$, is strictly positive, while the minimum value of $\tilde{V}_{B,u}$ when $\beta = 1$ equals $\left(1-\frac{\alpha_B}{\alpha_A}\right)(1-2\alpha_B)$. Thus, when $\alpha_B \leq 1/2$, we have $\tilde{V}_{B,u}|_{\beta=1} \geq 0$, so $\tilde{V}_{B,u} \geq 0$ for any $\beta \in (0,1)$. When $\alpha_B > 1/2$, we have $\tilde{V}_{B,u}|_{\beta=1} < 0$. Then, there exists a unique threshold $\bar{\beta} \in (0,1)$, such that $\tilde{V}_{B,u}$ is negative if and only if $\beta > \bar{\beta}$. □

Proof of Lemma 3. From (17) and (18), after a simple calculation, we get

$$V_{A,u} - V_{B,u} = \begin{cases} (1-\beta)[\alpha_A - \alpha_B(1-(1+\beta)\alpha_B)] \text{ if } \beta \geq \frac{\alpha_B}{\alpha_A} \\ \alpha_B\beta(\beta+1)(\alpha_A-\alpha_B) \text{ if } \beta < \frac{\alpha_B}{\alpha_A} \end{cases},$$

which is always positive. □

Proofs of Lemma 4 and Lemma 5.. Lemma 4 and Lemma 5 can be easily drawn from Lemma 1. □

Proof of Proposition 2. We define $g(\beta) = V_{tp} - V_{B,u}$. Recall that, under TP, the total auction payoff for the publisher is as follows:

$$V_{tp} = \begin{cases} \alpha_l(1-\beta)[1-(1+\beta)\alpha_l] \text{ if } \beta \geq \frac{\alpha_l}{\alpha_h} \\ \alpha_l\left[1-(1+\alpha_h)\beta+\beta^2\alpha_l\right] \text{ if } \beta < \frac{\alpha_l}{\alpha_h} \end{cases}. \tag{19}$$

From (18) and (19), we have

$$g(\beta) = \begin{cases} \alpha_l(1-\beta)[1-(1+\beta)\alpha_l] - \alpha_B(1-\beta)[1-(1+\beta)\alpha_B] \text{ if } \beta \geq \frac{\alpha_B}{\alpha_A} \\ \alpha_l(1-\beta)[1-(1+\beta)\alpha_l] - \alpha_B\left[1-\alpha_A\beta-\beta+\beta^2\alpha_B\right] \text{ if } \frac{\alpha_l}{\alpha_h} < \beta < \frac{\alpha_B}{\alpha_A} \\ \alpha_l\left[1-\alpha_h\beta-\beta+\beta^2\alpha_l\right] - \alpha_B\left[1-\alpha_A\beta-\beta+\beta^2\alpha_B\right] \text{ if } \beta \leq \frac{\alpha_l}{\alpha_h} \end{cases}.$$

Note that $g(\beta)$ is a continuous function of β on $[0,1]$.

Recall that $\alpha_B = \gamma\alpha_l + (1-\gamma)\alpha_h$ and $\alpha_A = \gamma\alpha_h + (1-\gamma)\alpha_l$. For $\beta \leq \frac{\alpha_l}{\alpha_h}$, we have

$$\begin{aligned} g(\beta) &\leq \alpha_B\left[\left(1-\alpha_h\beta-\beta+\beta^2\alpha_l\right) - \left(1-\alpha_A\beta-\beta+\beta^2\alpha_B\right)\right] \\ &= \alpha_B\left[\beta(\alpha_A-\alpha_h) + \beta^2(\alpha_l-\alpha_B)\right] < 0. \end{aligned}$$

In particular, we have $g(\beta)|_{\beta=\frac{\alpha_l}{\alpha_h}} < 0$.

For $\frac{\alpha_l}{\alpha_h} < \beta < \frac{\alpha_B}{\alpha_A}$, we have

$$\begin{aligned} g(\beta) &= \alpha_l(1-\beta)[1-(1+\beta)\alpha_l] - \alpha_B\left[1-\alpha_A\beta-\beta+\beta^2\alpha_B\right] \\ &= (1-\beta)\{\frac{\alpha_B(\alpha_A-\alpha_B)}{1-\beta} + \left((1+\beta)\left(\alpha_B^2-\alpha_l^2\right) - (\alpha_B\alpha_A+\alpha_B-\alpha_l)\right)\}. \end{aligned}$$

For $\beta \geq \frac{\alpha_B}{\alpha_A}$, we have

$$\begin{aligned} g(\beta) &= \alpha_l(1-\beta)[1-(1+\beta)\alpha_l] - \alpha_B(1-\beta)[1-(1+\beta)\alpha_B] \\ &= (1-\beta)(\alpha_B-\alpha_l)((1+\beta)(\alpha_l+\alpha_B)-1). \end{aligned}$$

Define

$$h(\beta) = \begin{cases} (\alpha_B-\alpha_l)((1+\beta)(\alpha_l+\alpha_B)-1) \text{ if } \beta \geq \frac{\alpha_B}{\alpha_A} \\ \frac{\alpha_B(\alpha_A-\alpha_B)}{1-\beta} + \left((1+\beta)\left(\alpha_B^2-\alpha_l^2\right) - (\alpha_B\alpha_A+\alpha_B-\alpha_l)\right) \text{ if } \frac{\alpha_l}{\alpha_h} < \beta < \frac{\alpha_B}{\alpha_A} \end{cases}.$$

Since $g(\beta) = (1-\beta)h(\beta)$ for all $\frac{\alpha_l}{\alpha_h} < \beta < 1$, $g(\beta)$ and $h(\beta)$ have the same sign. Furthermore, it is easy to observe that on the interval $[\frac{\alpha_l}{\alpha_h}, 1]$, $h(\beta)$ is continuous and increases in β. Since $g(\beta)|_{\beta=\frac{\alpha_l}{\alpha_h}} < 0$, we know that $h(\beta)|_{\beta=\frac{\alpha_l}{\alpha_h}} < 0$. Note that $h(\beta)|_{\beta=1} = (\alpha_B - \alpha_l)(2(\alpha_l + \alpha_B) - 1)$, which is negative if and only if $\alpha_l + \alpha_B \leq 1/2$.

Therefore, we have:

i) If $\alpha_l + \alpha_B \leq 1/2$, then $h(\beta) \leq 0$ for $\beta \in [\frac{\alpha_l}{\alpha_h}, 1]$. This implies $g(\beta) \leq 0$ for $\beta \in [0,1]$;
ii) If $\alpha_l + \alpha_B \geq 1/2$, then there exists $\hat{\beta} \in (\frac{\alpha_l}{\alpha_h}, 1]$ such that $g(\beta) \leq 0$ for $\beta \in \left[0, \hat{\beta}\right]$ and $g(\beta) > 0$ for $\beta \in (\hat{\beta}, 1]$.

Since $\alpha_B = \gamma\alpha_l + (1-\gamma)\alpha_h$, we have $\alpha_l + \alpha_B \leq 1/2 \Leftrightarrow \gamma \geq \frac{\alpha_h + \alpha_l - 1/2}{\alpha_h - \alpha_l}$. Note that $\gamma \in [1/2, 1)$. We define:

$$\hat{\gamma}_0 = \begin{cases} 1/2 \text{ if } \frac{\alpha_h+\alpha_l-1/2}{\alpha_h-\alpha_l} < 1/2 \\ \frac{\alpha_h+\alpha_l-1/2}{\alpha_h-\alpha_l} \text{ if } 1/2 \leq \frac{\alpha_h+\alpha_l-1/2}{\alpha_h-\alpha_l} \leq 1 \\ 1 \text{ if } \frac{\alpha_h+\alpha_l-1/2}{\alpha_h-\alpha_l} > 1 \end{cases},$$

such that the threshold value $\hat{\gamma}_0$ is always in the interval $[1/2, 1]$ and satisfies $\alpha_l + \alpha_B \leq 1/2$ if and only if $\gamma \in [\hat{\gamma}_0, 1)$. □

Proof of Proposition 3. We denote by Π_{tp} and Π_u the industry profit in the product market under TP and UA respectively. Then,

$$\Pi_{tp} = \alpha_h(1-\beta\alpha_l) + \beta\alpha_l(1-\beta\alpha_l), \tag{20}$$

$$\Pi_u = \alpha_A(1-\beta\alpha_B) + \beta\alpha_B(1-\beta\alpha_B), \tag{21}$$

Note that Π_u is a function of γ and $\Pi_{tp} = \Pi_u|_{\gamma=1}$.

$$\frac{\partial \Pi_u}{\partial \gamma} = (\alpha_h - \alpha_l)(1 - \beta + \beta(\alpha_A - \alpha_B + 2\beta\alpha_B)) > 0.$$

Therefore, industry profit is higher under targeted advertising.

Denote $\pi_{i,tp}$, $\pi_{i,u}$ as firm i's profits in the product markets, and $\hat{\pi}_{i,tp}$, $\hat{\pi}_{i,u}$ as its net profits under targeted and UA, respectively. In position auctions, the high type firm always wins, and pays the low type firm's valuation of the prominent position, while the low type firm loses and does not pay anything. We have $\hat{\pi}_{B,u} = \pi_{B,u}$, $\hat{\pi}_{A,u} = \pi_{A,u} - V_{B,u}$, $\hat{\pi}_{A,tp} = \gamma\left(\pi^i_{i,tp} - V^i_{j,tp}\right) + (1-\gamma)\pi^i_{j,tp}$, and $\hat{\pi}_{B,tp} = (1-\gamma)\left(\pi^i_{i,tp} - V^i_{j,tp}\right) + \gamma\pi^i_{j,tp}$.

$$\begin{aligned} \hat{\pi}_{B,tp} - \hat{\pi}_{B,u} &= (1-\gamma)\left(\pi^i_{i,tp} - V^i_{j,tp}\right) + \gamma\pi^i_{j,tp} - \pi_{B,u} \\ &= \begin{cases} (1-\gamma)(\alpha_h(1-\beta\alpha_l) - \alpha_l(1-\beta)(1-(1+\beta)\alpha_l)) \\ \quad + \gamma\beta\alpha_l(1-\beta\alpha_l) - \beta\alpha_B(1-\beta\alpha_B) \text{ if } \beta \geq \frac{\alpha_l}{\alpha_h} \\ (1-\gamma)\left(\alpha_h(1-\beta\alpha_l) - \alpha_l\left[1 - \alpha_h\beta - \beta + \beta^2\alpha_l\right]\right) \\ \quad + \gamma\beta\alpha_l(1-\beta\alpha_l) - \beta\alpha_B(1-\beta\alpha_B) \text{ if } \beta < \frac{\alpha_l}{\alpha_h} \end{cases}. \end{aligned}$$

Note that $\alpha_B = \gamma\alpha_l + (1-\gamma)\alpha_h > \alpha_l$. Thus, for $\beta \geq \frac{\alpha_l}{\alpha_h}$, we have:

$$\begin{aligned} \hat{\pi}_{B,tp} - \hat{\pi}_{B,u} &= (1-\gamma)(\alpha_h((1-\beta\alpha_l) - \beta(1-\beta\alpha_B)) - \alpha_l(1-\beta)(1-(1+\beta)\alpha_l)) \\ &\quad + \gamma\alpha_l(\beta(1-\beta\alpha_l) - \beta(1-\beta\alpha_B)) \\ &\geq (1-\gamma)(\alpha_h(1-\beta\alpha_l)(1-\beta) - \alpha_l(1-\beta)(1-(1+\beta)\alpha_l)) \\ &\geq (1-\gamma)(\alpha_l(1-\beta\alpha_l)(1-\beta) - \alpha_l(1-\beta)(1-(1+\beta)\alpha_l)) \\ &= (1-\gamma)(1-\beta)\alpha_l^2 > 0. \end{aligned}$$

Similarly, for $\beta < \frac{\alpha_l}{\alpha_h}$, we have:

$$\begin{aligned} \hat{\pi}_{B,tp} - \hat{\pi}_{B,u} &= (1-\gamma)\left(\alpha_h((1-\beta\alpha_l) - \beta(1-\beta\alpha_B)) - \alpha_l\left(1 - \alpha_h\beta - \beta + \beta^2\alpha_l\right)\right) \\ &\quad + \gamma\alpha_l(\beta(1-\beta\alpha_l) - \beta(1-\beta\alpha_B)) \\ &\geq (1-\gamma)\left(\alpha_h((1-\beta\alpha_l) - \beta(1-\beta\alpha_l)) - \alpha_l\left(1 - \alpha_h\beta - \beta + \beta^2\alpha_l\right)\right) \\ &= (1-\gamma)(\alpha_h - \alpha_l)\left(1 - \beta + \beta^2\alpha_l\right) > 0. \end{aligned}$$

Thus, the low type firm always prefers targeted advertising.

For the high type firm A, we have:

$$\hat{\pi}_{A,tp} - \hat{\pi}_{A,u} = \gamma\left(\pi^i_{i,tp} - V^i_{j,tp}\right) + (1-\gamma)\pi^i_{j,tp} - (\pi_{A,u} - V_{B,u}).$$

For $\beta < \frac{\alpha_l}{\alpha_h}$, we have

$$\hat{\pi}_{A,tp} - \hat{\pi}_{A,u} = (1-\gamma)(\alpha_h - \alpha_l)\left(1 - \beta + (\alpha_l + \alpha_B)\beta^2\right) > 0.$$

Furthermore, it follows from the expressions of $V^i_{j,tp}$ and $V_{B,u}$ that if $\frac{\alpha_l}{\alpha_h} < \beta \le \frac{\alpha_B}{\alpha_A}$, then we have

$$\hat{\pi}_{A,tp} - \hat{\pi}_{A,u} = (1-\gamma)(\alpha_h - \alpha_l)\left(1 - \beta + (\alpha_l + \alpha_B)\beta^2\right) + \gamma\alpha_l^2 - \gamma\alpha_l\alpha_h\beta,$$

and if $\beta > \frac{\alpha_B}{\alpha_A}$,then we have

$$\hat{\pi}_{A,tp} - \hat{\pi}_{A,u} = (1-\gamma)(\alpha_h - \alpha_l)\left(1 - \beta + (\alpha_l + \alpha_B)\beta^2\right) + \gamma\alpha_l^2 - \alpha_B^2 - \gamma\alpha_l\alpha_h\beta + \alpha_B\alpha_A\beta.$$

Note that $\hat{\pi}_{A,tp} - \hat{\pi}_{A,u}$ is a convex function of β on $\left[\frac{\alpha_l}{\alpha_h}, 1\right]$. Furthermore, it is easy to verify that $\hat{\pi}_{A,tp} - \hat{\pi}_{A,u}|_{\beta=1} = (\alpha_h - \alpha_l)((1-\gamma)\alpha_l + \gamma(\alpha_B - \alpha_l)) > 0$. Thus, if $\min_{\beta\in\left[\frac{\alpha_l}{\alpha_h},1\right]} \hat{\pi}_{A,tp} - \hat{\pi}_{A,u} < 0$, then there exist two threshold values $\hat{\beta}_1$ and $\hat{\beta}_2$, such that $\hat{\pi}_{A,tp} - \hat{\pi}_{A,u} \le 0$ for $\beta \in \left[\hat{\beta}_1, \hat{\beta}_2\right]$, and $\hat{\pi}_{A,tp} - \hat{\pi}_{A,u} > 0$ for $\beta \in [0, \hat{\beta}_1) \cup (\hat{\beta}_2, 1]$. Note that when $\gamma = 1$ and $\beta \in \left(\frac{\alpha_l}{\alpha_h}, \frac{\alpha_B}{\alpha_A}\right)$, we have $\hat{\pi}_{A,tp} - \hat{\pi}_{A,u} < 0$. Thus, there exist cases in which $\min_{\beta\in\left[\frac{\alpha_l}{\alpha_h},1\right]} \hat{\pi}_{A,tp} - \hat{\pi}_{A,u} < 0$. □

Proof of Proposition 4.

1) Shoppers always search for two firms. The expected utility of type A shoppers under UA is:

$$U_{A,u} = \alpha_h(1-\alpha_l)(1 - E\tilde{p}_{A,u}) + \alpha_l(1-\alpha_h)(1 - E\tilde{p}_{B,u}) + \alpha_l\alpha_h(1 - E\min(\tilde{p}_{A,u}, \tilde{p}_{B,u}))$$

and his/her expected utility under targeted advertising is:

$$U_{A,tp} = \alpha_h(1-\alpha_l)\left(1 - E\tilde{p}^A_{A,tp}\right) + \alpha_l(1-\alpha_h)\left(1 - E\tilde{p}^A_{B,tp}\right) + \alpha_l\alpha_h\left(1 - E\min\left(\tilde{p}^A_{A,tp}, \tilde{p}^A_{B,tp}\right)\right).$$

It follows from Lemma 5 that $\tilde{p}^A_{A,tp}$ first-order stochastically dominates $\tilde{p}_{A,u}$ and $\tilde{p}^A_{B,tp}$ first-order stochastically dominates $\tilde{p}_{B,u}$. Therefore, $U_{A,u} \ge U_{A,tp}$.

Similarly, we can show that a type B shopper prefers UA.

Now, we consider nonshoppers. Under UA, product A wins and appears prominently to all consumers. The expected utility of type A nonshoppers is:

$$U_{A,u} = \alpha_h(1 - E\tilde{p}_{A,u}) = \alpha_h\left(\int_{1-\alpha_B}^1 (1-p)\frac{1-\alpha_B}{\alpha_A}\frac{1}{p^2}dp\right)$$

and the expected utility of type B nonshoppers is

$$U_{B,u} = \alpha_l(1 - E\tilde{p}_{A,u}) = \alpha_l\left(\int_{1-\alpha_B}^1 (1-p)\frac{1-\alpha_B}{\alpha_A}\frac{1}{p^2}dp\right).$$

Under targeted advertising, product i wins the prominence with type i consumers.Thus, nonshoppers gain regardless of their preference for the product:

$$U_{i,tp} = \alpha_h\left(1 - E\tilde{p}^i_{i,u}\right) = \alpha_h\left(\int_{1-\alpha_l}^1 (1-p)\frac{1-\alpha_l}{\alpha_h}\frac{1}{p^2}dp\right),\ i = A, B.$$

Since $\tilde{p}^i_{i,u}$ first-order stochastically dominates $\tilde{p}_{A,u}$, we have $U_{A,u} > U_{i,tp}$.

$$U_{B,u} - U_{i,tp} = \alpha_l(1 - E\tilde{p}_{A,u}) - \alpha_h\left(1 - E\tilde{p}^i_{i,u}\right).$$

We define $x = 1 - \alpha_B$ and $f(x) = \int_x^1 (1-p)\frac{x}{p^2}dp = 1 - x + x\ln x$. Then, $f'(x) = \ln x < 0$.

$$\frac{\partial\left(U_{B,u} - U_{i,tp}\right)}{\partial\gamma} = \frac{\partial\frac{\alpha_l}{\alpha_A}}{\partial\gamma}f(x) - \frac{\alpha_l}{\alpha_A}f'(x)\frac{\partial\alpha_B}{\partial\gamma} < 0.$$

As $\gamma \to 1$, $1 - E\tilde{p}_{A,u} \to 1 - E\tilde{p}^i_{i,u}$, it follows that $U_{B,u} - U_{i,tp}|_{\gamma=1} < 0$.

Now, let us prove $U_{B,u}|_{\gamma=\frac{1}{2}} > U_{i,tp}$. Note that $U_{B,u}|_{\alpha_h=\alpha_l} = U_{i,tp}$. Thus, in order to prove $U_{B,u}|_{\gamma=\frac{1}{2}} > U_{i,tp}$, we only need to prove that $\frac{\partial U_{B,u}|_{\gamma=\frac{1}{2}}}{\partial a_h} > 0$. Note that

$$U_{B,u}|_{\gamma=\frac{1}{2}} = \frac{\alpha_l}{1-x}f(x).$$

We only need to prove $\frac{\partial U_{B,u}|_{\gamma=\frac{1}{2}}}{\partial x} < 0$, since $x = 1 - \frac{\alpha_h+\alpha_l}{2}$ is decreasing in α_h.

$$\begin{aligned}\frac{\partial U_{B,u}|_{\gamma=\frac{1}{2}}}{\partial x} &= \frac{\alpha_l}{(1-x)^2} f(x) + \frac{\alpha_l}{1-x} f'(x)\\ &= \frac{\alpha_l}{(1-x)^2}(1 - x + \ln x)\\ &= \frac{\alpha_l}{(1-x)^2}\left(\int_x^1 \left(1 - \frac{1}{p}\right) dp\right) < 0.\end{aligned}$$

Now, we have proved that $U_{B,u} - U_{i,tp}$ is decreasing in γ, with $U_{B,u}|_{\gamma=\frac{1}{2}} > U_{i,tp}$ and $U_{B,u}|_{\gamma=1} < U_{i,tp}$. Thus, there exists $\hat{\gamma}_1 \in (1/2, 1)$, such that type B is better off under UA if and only if $\gamma < \hat{\gamma}_1$.

Considering the total consumer surplus of nonshoppers, we have:

$$\begin{aligned}\gamma\left(U_{A,u} - U_{i,tp}\right) + (1-\gamma)\left(U_{B,u} - U_{i,tp}\right) &= (\gamma\alpha_h + (1-\gamma)\alpha_l)(1 - E\tilde{p}_{A,u}) - \alpha_h\left(1 - E\tilde{p}^i_{i,u}\right)\\ &= \alpha_A\left(\int_{(1-\alpha_B)}^1 (1-p)\frac{1-\alpha_B}{\alpha_A}\frac{1}{p^2}dp\right) - \alpha_h\left(\int_{1-\alpha_l}^1 (1-p)\frac{1-\alpha_l}{\alpha_h}\frac{1}{p^2}dp\right)\\ &= f(1-\alpha_B) - f(1-\alpha_l) > 0.\end{aligned}$$

Thus, the total consumer surplus of nonshoppers is higher under UA. Since shoppers are also strictly better off under UA, the total consumer surplus is higher under UA.

2) Define TS_{tp} and TS_u as the total surplus under TP and UA respectively. We have:

$$TS_{tp} = (1-\beta)\alpha_h + \beta(1 - (1-\alpha_h)(1-\alpha_l)), \tag{22}$$

$$TS_u = (1-\beta)\alpha_A + \beta(1 - (1-\alpha_h)(1-\alpha_l)). \tag{23}$$

It follows that $TS_{tp} > TS_u$. □

Proof of Proposition 5. The publisher's payoff under TS is $(1-\beta)\alpha_B$, which is higher than $(1-\beta)\alpha_l$, i.e., its payoff under T2. It is also easy to verify from (18) and (19) that $(1-\beta)\alpha_B$ is greater than $V_{B,u}$ and $V^j_{i,tp}$. □

Proof of Proposition 6.

1) Let Π_{ts} and Π_{t2} denote the industry profit under TS and T2 respectively. We have:

$$\begin{aligned}\Pi_{ts} - \Pi_u &= \beta\alpha_A(1-\alpha_B) + (1-\beta)\alpha_A - ((\alpha_A + \beta\alpha_B)(1-\beta\alpha_B))\\ &= -\beta\alpha_B(1-\beta\alpha_B) < 0,\end{aligned}$$

$$\begin{aligned}\Pi_{t2} - \Pi_{tp} &= \beta\alpha_h(1-\alpha_l) + (1-\beta)\alpha_h - (\alpha_h(1-\beta\alpha_l) + \beta\alpha_l(1-\beta\alpha_l))\\ &= -\beta\alpha_l(1-\beta\alpha_l) < 0,\end{aligned}$$

implying that TS reduces industry profit.

We also have:

$$\Pi_{t2} - \Pi_{ts} = \beta\alpha_h(1-\alpha_l) + (1-\beta)\alpha_h - (\beta\alpha_A(1-\alpha_B) + (1-\beta)\alpha_A),$$

$$\frac{\partial \Pi_{t2} - \Pi_{ts}}{\partial\gamma} = -(\alpha_h - \alpha_l)(\beta(1-\alpha_B) + \beta\alpha_A + (1-\beta)) < 0.$$

Note that $\Pi_{t2} - \Pi_{ts}|_{\gamma=1} = 0$. Therefore, $\Pi_{t2} - \Pi_{ts} > 0$. Furthermore, we already know from Proposition 3 that $\Pi_{tp} > \Pi_u$. Thus, TP increases industry profit, whether or not the publisher implements TS.

For firms, we have:

$$\pi_{B,u} - \pi_{B,ts} = \beta\alpha_B(1-\beta\alpha_B) - \beta\alpha_B(1-\alpha_B) > 0,$$

$$\begin{aligned}\pi_{B,tp} - \pi_{B,t2} &= \gamma\beta\alpha_l(1-\beta\alpha_l) + (1-\gamma)\left(\alpha_h(1-\beta\alpha_l) - V^j_{i,tp}\right)\\ &\quad - (\gamma\beta\alpha_l(1-\alpha_l) + (1-\gamma)(\beta\alpha_h(1-\alpha_l) + (1-\beta)(\alpha_h - \alpha_l)))\\ &= \gamma\beta\alpha_l(\alpha_l - \beta\alpha_l) + (1-\gamma)\left((1-\beta)\alpha_l - V^j_{i,tp}\right) > 0,\end{aligned}$$

where the last inequality is due to the fact that $(1-\beta)\alpha_l > V^j_{i,tp}$. Therefore, TS always hurts firm B.

Similarly, we have:

$$\begin{aligned}\hat{\pi}_{A,u} - \hat{\pi}_{A,ts} &= \alpha_A(1-\beta\alpha_B) - V_{B,u} - (\beta\alpha_A(1-\alpha_B) + (1-\beta)(\alpha_A-\alpha_B))\\ &= (1-\beta)\alpha_B - V_{B,u} > 0\end{aligned}$$

and

$$\hat{\pi}_{A,tp} - \hat{\pi}_{A,t2} = (1-\gamma)\beta(1-\beta)\alpha_l^2 + \gamma\left((1-\beta)\alpha_l - V_{i,tp}^j\right) > 0,$$

where $\hat{\pi}_{A,t2} = \beta\gamma\alpha_h(1-\alpha_l) + \beta(1-\gamma)\alpha_l(1-\alpha_l) + (1-\beta)\gamma(\alpha_h-\alpha_l)$.

Therefore, TS always hurts firm A.

Finally, we have:

$$\pi_{B,t2} - \pi_{B,ts} = (1-\beta)(1-\gamma)(\alpha_h-\alpha_l) + \beta\alpha_B(\alpha_B-\alpha_l) > 0.$$

This means that TP always benefits firm B, regardless of whether the publisher has implemented TS or not.

2) The results for nonshoppers are obvious, because nonshoppers get zero utility if the publisher implements TS. The results for shoppers are also straightforward, because TP eases price competition and therefore shoppers pay lower prices.

Now, we consider the total consumer surplus:

$$\begin{aligned}CS_u - CS_{ts} &= [(\beta(1-(1-\alpha_A)(1-\alpha_B)) + (1-\beta)\alpha_A) - \Pi_u] - [(\beta(1-(1-\alpha_A)(1-\alpha_B)) + (1-\beta)\alpha_A) - \Pi_{ts}]\\ &= \Pi_{ts} - \Pi_u < 0,\end{aligned}$$

$$\begin{aligned}CS_{tp} - CS_{t2} &= [(\beta(1-(1-\alpha_A)(1-\alpha_B)) + (1-\beta)\alpha_h)v - \Pi_{tp}] - [(\beta(1-(1-\alpha_A)(1-\alpha_B)) + (1-\beta)\alpha_h)v - \Pi_{t2}]\\ &= \Pi_{t2} - \Pi_{tp} < 0.\end{aligned}$$

Thus, TS increases the total consumer surplus.

We also have:

$$\begin{aligned}CS_{ts} - CS_{t2} &= \Pi_{t2} - \Pi_{ts} + (1-\beta)(\alpha_A-\alpha_h)\\ &= \beta\alpha_h(1-\alpha_l) + (1-\beta)\alpha_h - (\beta\alpha_A(1-\alpha_B) + (1-\beta)\alpha_A) + (1-\beta)(\alpha_A-\alpha_h)\\ &= \beta(\alpha_h(1-\alpha_l) - \alpha_A(1-\alpha_B)) = (1-\gamma)(\alpha_h-\alpha_l)(\gamma(\alpha_h-\alpha_l)+1)\\ &> 0.\end{aligned}$$

In other words, TP always reduces consumer surplus, regardless of whether the publisher has implemented TS or not.

3) Total surplus depends only on the trading probability. TS does not change the advertisement displayed prominently to each consumer and, therefore, does not change the trading probability. □

Proof of Corollary 1. Corollary 1 follows immediately from the previous results. □

Proof of Proposition 7. We prove the equivalence of UA and TS, and the proof of the equivalence of TP and T2 is similar.

Suppose that under UA, firm i wins the prominent position. Under TS, firm i wins the prominent position facing nonshoppers (it does not matter who wins the prominent position to shoppers since shoppers always search for both firms). Then, the demand function for the firms under UA is given by (14) and (15). Now, we consider the demand functions for the two firms under TS. Shoppers always buy from the lower priced firm, while nonshoppers buy only from firm i. If $p_i < p_j$, then firm i's demand is α_i and firm j's demand is $\beta\alpha_j(1-\alpha_i)$; if $p_i > p_j$, then, firm i's demand is $\beta\alpha_i\left(1-\alpha_j\right) + (1-\beta)\alpha_i = \alpha_i\left(1-\beta\alpha_j\right)$ and firm j's demand is $\beta\alpha_j$; if $p_i = p_j$, then firm i's demand is $\beta\left(\alpha_i\left(1-\alpha_j\right) + \frac{\alpha_i\alpha_j}{2}\right) + (1-\beta)\alpha_i = \alpha_i\left(1-\frac{\beta\alpha_j}{2}\right)$ and firm j's demand is $\beta\left(\alpha_j(1-\alpha_i) + \frac{\alpha_i\alpha_j}{2}\right) = \beta\alpha_j\left(1-\frac{\alpha_i}{2}\right)$. The demand functions for the two firms are the same as (14) and (15). Therefore, the firms' pricing strategies under UA are the same as those under TS. □

Proof of Lemma 6. Assume that firm A wins type A market in the first auction. Conditional on A winning the first auction, firm A will bid

$$V_{A|A} = \pi_u^h - \pi_1$$

and firm B will bid

$$V_{B|A} = \pi_1 - \pi_u^l.$$

Firm A outbids firm B if and only if $V_{A|A} \geq V_{B|A}$, which is equivalent to $\Pi_u \geq \Pi_1$. After a simple calculation, we have:

$$\Pi_u - \Pi_1 = \frac{1}{2}(1-\beta)\left[\alpha_l - \alpha_h + \alpha_h^2 + \beta\left(\alpha_h^2 - \frac{1}{2}(\alpha_h-\alpha_l)^2\right)\right],$$

which can be either positive or negative, depending on the parameters.[13]

Now, assume firm B wins type A market in the first auction. In the second auction, firm A will bid

$$V_{A|B} = \pi_2 - \pi_u^l$$

and firm B will bid

$$V_{B|B} = \pi_u^h - \pi_2.$$

Firm B outbids firm A in the second auction if and only if $V_{B|B} \geq V_{A|B}$, which is equivalent to $\Pi_u \geq \Pi_2$. After simple calculations, we have:

$$\Pi_u - \Pi_2 = \frac{1}{4}(1-\beta)\Big((\alpha_h - \alpha_l)(2 - \beta(\alpha_h - \alpha_l)) + 2\alpha_l^2(1+\beta)\Big) \geq 0,$$

which is positive. Conditional on winning its weak market in the first auction, firm B always outbids firm A for its strong market in the second auction. □

Proof of Lemma 7. We have two cases:

Case 1: $\Pi_u \geq \Pi_1$. In this case, whoever wins the first auction wins both markets. In the first auction, A will bid

$$V_A = \pi_u^h - V_{B|A} - \pi_u^l$$

and B will bid

$$V_B = \pi_u^h - V_{A|B} - \pi_u^l.$$

Firm A outbids firm B if $V_A - V_B \geq 0$. We have $V_A - V_B = V_{A|B} - V_{B|A} = \pi_2 - \pi_1$, which can be either positive or negative. Therefore, in the first auction, either firm A or firm B can win, depending on the parameters.

Case 2: $\Pi_u \leq \Pi_1$. Then, regardless of who wins the first auction, B always wins the second auction. In the first auction, A will bid

$$V_A = \pi_1 - \pi_u^l$$

and B will bid

$$V_B = \pi_u^h - V_{A|B} - \pi_1.$$

A wins the first auction if $V_A \geq V_B$, which is equivalent to

$$\Pi_1 \geq \Pi_u - V_{A|B}.$$

This holds because $\Pi_u \leq \Pi_1$. Therefore, in this case, A always wins the first auction. □

Proof of Proposition 8. If $\Pi_u \geq \Pi_1$, the publisher's auction payoff is equal to $V_B + V_{B|A} = \pi_u^h - \pi_u^l + V_{B|A} - V_{A|B} < \pi_u^h - \pi_u^l$ if A wins the first auction, and equal to $V_A + V_{A|B} = \pi_u^h - \pi_u^l + V_{A|B} - V_{B|A} < \pi_u^h - \pi_u^l$ if B wins the first auction. Note that $\pi_u^h - \pi_u^l$ is the publisher's auction payoff under uniform pricing. The publisher strictly prefers UA.

If $\Pi_u \leq \Pi_1$, the publisher's auction payoff is equal to $V_B + V_{A|A} < V_A + V_{A|A} = \pi_1 - \pi_u^l + \pi_u^h - \pi_1 = \pi_u^h - \pi_u^l$. Therefore, the publisher prefers UA. □

Proof of Proposition 9. Substituting the expressions for $\alpha_{\sigma_i}^i$ and $\alpha_{\sigma_i}^j$ into the expression for V, we get:

$$V = \begin{cases} V_1 \text{ if } a \geq \hat{a}_0 \\ V_2 \text{ if } 1/2 \leq a \leq \hat{a}_0 \end{cases},$$

where

$$V_1 = (1-\beta)(\alpha_h - (\alpha_h - \alpha_l)a)[1 - (1+\beta)\alpha_h + (1+\beta)(\alpha_h - \alpha_l)a]$$
$$V_2 = (\alpha_h - (\alpha_h - \alpha_l)a)\Big[1 - \beta - \alpha_l\beta + \beta^2\alpha_h - \big(\beta + \beta^2\big)(\alpha_h - \alpha_l)a\Big]$$

and $\hat{a}_0 = \frac{\alpha_h - \beta\alpha_l}{(\alpha_h - \alpha_l)(1+\beta)}$, which satisfies $\beta = \frac{\alpha_{\sigma_i}^j}{\alpha_{\sigma_i}^i} = \frac{\hat{a}_0\alpha_l + (1-\hat{a}_0)\alpha_h}{\hat{a}_0\alpha_h + (1-\hat{a}_0)\alpha_l}$.

Note that V_2 is a quadratic and convex function of a and therefore reaches a maximum at $a = 1/2$ or $a = \hat{a}_0$. Furthermore, V_1 is a quadratic and concave function of a. Solving $\frac{\partial V_1}{\partial a} = 0$ gives:

$$\hat{a}_1 = \frac{2(1+\beta)\alpha_h - 1}{2(\alpha_h - \alpha_l)(1+\beta)}.$$

[13] In particular, if $\alpha_l > \alpha_h(1-\alpha_h)$, then $\Pi_u - \Pi_1 \geq 0$ for all $\beta \in [0,1]$; if $\alpha_l < \alpha_h + \frac{1}{2}(\alpha_h - \alpha_l)^2 - 2\alpha_h^2$, then $\Pi_u - \Pi_1 \leq 0$ for all $\beta \in [0,1]$; if $\alpha_h + \frac{1}{2}(\alpha_h - \alpha_l)^2 - 2\alpha_h^2 < \alpha_l < \alpha_h(1-\alpha_h)$, then $\Pi_u - \Pi_1 \geq 0$ if $\beta > \frac{\alpha_h(1-\alpha_h) - \alpha_l}{\alpha_h^2 - \frac{1}{2}(\alpha_h - \alpha_l)^2}$ and $\Pi_u - \Pi_1 \leq 0$ if $\beta < \frac{\alpha_h(1-\alpha_h) - \alpha_l}{\alpha_h^2 - \frac{1}{2}(\alpha_h - \alpha_l)^2}$.

According to the expression for V, the optimal a^* should satisfy $a^* \in \{1/2, \hat{a}_0, \hat{a}_1, 1\}$.

It is easy to calculate:

$$V(1/2) = \frac{1}{2}t(1-\beta)\left(1 - \frac{1}{2}\beta t\right), \tag{24}$$

$$V(\hat{a}_0) = \frac{\beta}{1+\beta}t(1-\beta)(1-\beta t),$$

$$V(\hat{a}_1) = \frac{1-\beta}{4(1+\beta)}, \tag{25}$$

where $t = \alpha_h + \alpha_l$.

Since $\frac{1}{2} > \frac{\beta}{1+\beta}$ and $1 - \frac{1}{2}\beta t > 1 - \beta t$, we have $V(1/2) > V(\hat{a}_0)$. Therefore, it must be the case that $a^* \in \{1/2, \hat{a}_1, 1\}$.

In what follows, we prove our results by contradiction. Suppose $a^* = \hat{a}_1$. We must have $\hat{a}_0 \le \hat{a}_1 < 1$, which implies that

$$\frac{\alpha_h - \beta\alpha_l}{(\alpha_h - \alpha_l)(1+\beta)} \le \frac{2(1+\beta)\alpha_h - 1}{2(\alpha_h - \alpha_l)(1+\beta)} < 1$$

$\Rightarrow$

$$\frac{1}{2(\alpha_h + \alpha_l)} \le \beta < \frac{1}{2\alpha_l} - 1 \tag{26}$$

Since $0 < \beta < 1$, in order for condition (26) to hold, we must have:

$$\alpha_l < 1/2 \text{ and } \alpha_h + \alpha_l > 1/2$$

$\Rightarrow$

$$1/2 < t < 3/2, \text{ where } t = \alpha_h + \alpha_l. \tag{27}$$

From (24) and (25), we have:

$$\begin{aligned} V(1/2) - V(\hat{a}_1) &= \frac{1-\beta}{4(1+\beta)}[t(1+\beta)(2-\beta t) - 1] \\ &= \frac{1-\beta}{4(1+\beta)} f(\beta; t), \end{aligned}$$

where $f(\beta; t) = t(1+\beta)(2-\beta t) - 1$.

Note that due to condition (27), we have:

$$f|_{\beta=0} = 2t - 1 > 0,$$

$$f|_{\beta=1} = -2(t-1)^2 - 1 > 0,$$

Since $f(\beta; t)$ is a concave function of β for any given t, we must have $f(\beta; t) > 0$ for all $\beta \in (0, 1)$ whenever condition (27) holds. This contradicts the previous assumption that $a^* = \hat{a}_1$. □

Proof of Proposition 10.

1) $\frac{\alpha_B}{\alpha_A} \le \frac{1}{2}$. The publisher's auction payoff is:

$$V = \frac{\alpha_B}{2}[(1-b)(1-(1+b)\alpha_B) + b(1-(2-b)\alpha_B)],$$

if $b \le 1 - \frac{\alpha_B}{\alpha_A}$; and

$$V = \frac{\alpha_B}{2}\left[(1-b)(1-(1+b)\alpha_B) + \left(1 - (1+\alpha_A)(1-b) + \alpha_B(1-b)^2\right)\right],$$

if $b \ge 1 - \frac{\alpha_B}{\alpha_A}$.

Therefore, we have

$$\frac{\partial V}{\partial b} = \begin{cases} \alpha_B^2(2b-1) > 0 \text{ if } b \le 1 - \frac{\alpha_B}{\alpha_A} \\ \frac{1}{2}\alpha_B[2\alpha_B(2b-1) + \alpha_A] > 0 \text{ if } b \ge 1 - \frac{\alpha_B}{\alpha_A} \end{cases}.$$

2) $\frac{\alpha_B}{\alpha_A} \ge \frac{1}{2}$. The publisher's auction payoff is

$$V = \frac{\alpha_B}{2}\left[1 - (1+\alpha_A)b + \alpha_B b^2 + 1 - (1+\alpha_A)(1-b) + \alpha_B(1-b)^2\right],$$

if $b \le \frac{\alpha_B}{\alpha_A}$; and

$$V = \frac{\alpha_B}{2}\left[(1-b)(1-(1+b)\alpha_B) + 1 - (1+\alpha_A)(1-b) + \alpha_B(1-b)^2\right],$$

if $b \geq \frac{\alpha_B}{\alpha_A}$.

Therefore, we have

$$\frac{\partial V}{\partial b} = \begin{cases} \alpha_B^2(2b-1) > 0 \text{ if } b \leq \frac{\alpha_B}{\alpha_A} \\ \frac{1}{2}\alpha_B[2\alpha_B(2b-1)+\alpha_A] > 0 \text{ if } b \geq \frac{\alpha_B}{\alpha_A} \end{cases}.$$

□

References

Amaldoss, W., Desai, P.S., Shin, W., 2015. Keyword search advertising and first-page bid estimates: a strategic analysis. Manage. Sci. 61 (3), 507–519.
Anderson, E.T., Simester, D.I., 2010. Price stickiness and customer antagonism. Q. J. Econ. 125 (2), 729–765.
Anderson, S., Renault, R., 2015. Search direction. Working Paper.
Angwin, J., 2010. The web's new gold mine: your secrets. Wall Street J.. July 30
Armstrong, M., Vickers, J., Zhou, J., 2009. Prominence and consumer search. Rand. J. Econ. 40 (2), 209–233.
Armstrong, M., Zhou, J., 2011. Paying for prominence. Econ. J. 121 (556), 368–395.
Athey, S., Ellison, G., 2011. Position auctions with consumer search. Q. J. Econ. 126 (3), 1213–1270.
Bazilian, E., 2011. Google Completes Rollout of Interest-Based Advertising: Adwords Will Be Able to Target Customers Based on Web Behavior. AdWeek technical report. June 28
Bergemann, D., Bonatti, A., 2011. Targeting in advertising markets: implications for offline versus online media. RAND J. Econ. 42 (3), 417–443.
Brahim, N., Lahmandi-Ayed, R., Laussel, D., 2011. Is targeted advertising always beneficial? Int. J. Ind Org. 29 (6), 678–689.
Chen, J., Liu, D., Whinston, A.B., 2009. Auctioning keywords in online search. J. Mark. 73 (4), 125–141.
Chen, J., Stallaert, J., 2014. An economics analysis of online advertising using behavioral targeting. MIS Q. 38 (2), 429–449.
Chen, Y., He, C., 2011. Paid placement: advertising and search on the internet. Econ. J. 121 (556), 309–328.
Chen, Y., Pearcy, J., 2010. Dynamic pricing: when to entice brand switching and when to reward consumer loyalty. Rand J. Econ. 41 (4), 674–685.
Childers, T.L., Carr, C.L., Peck, J., Carson, S., 2001. Hedonic and utilitarian motivations for online retail shopping behavior. J. Retail. 77 (4), 511–535.
Edelman, B., Ostrovsky, M., Schwarz, M., 2007. Internet advertising and the generalized second price auction: selling billions of dollars worth of keywords. Am. Econ. Rev. 97 (1), 242–259.
Esteves, R.B., Resende, J., 2016. Competitive targeted advertising with price discrimination. Market. Sci. 35 (4), 576–587.
Fishman, A., Lubensky, D., 2018. Search prominence and return costs. Int. J. Ind. Org. 58, 136–161.
Fudenberg, D., Tirole, J., 2000. Customer poaching and brand switching. Rand J. Econ. 31 (4), 634–657.
Fudenberg, D., Villas-Boas, J.M., 2006. Behavior-based Price Discrimination and Customer Recognition. In: Hendershott, T. (Ed.), Handbook on Economics and Information Systems. North-Holland, Amsterdam, pp. 377–436.
Gal-Or, E., Gal-Or, M., 2005. Customized advertising via a common media distributor. Market. Sci. 24 (2), 241–253.
Gal-Or, E., Gal-Or, M., May, J.H., Spangler, W.E., 2006. Targeted advertising strategies on television. Manage. Sci. 52 (5), 713–725.
Galeotti, A., Moraga-Gonzalez, J.L., 2008. Segmentation, advertising and prices. Int. J. Ind. Org. 26 (5), 1106–1119.
Ghose, A., Yang, S., 2009. An empirical analysis of search engine advertising: sponsored search in electric markets. Manage. Sci. 55 (10), 1605–1622.
Haan, M.A., Moraga-Gonzalez, J.L., 2011. Advertising for attention in a consumer search model. Econ. J. 121 (552), F552–F579.
Iyer, G., Soberman, D., Villas-Boas, J.M., 2005. The targeting of advertising. Market. Sci. 24 (3), 461–476.
Jerath, K., Ma, L., Park, Y.H., Srinivasan, K., 2011. A 'position paradox' in sponsored search auctions. Market. Sci. 30 (4), 612–627.
Johnson, J.P., 2013. Targeted advertising and advertising avoidance. Rand J. Econ. 44 (1), 128–144.
Katona, Z., Sarvary, M., 2010. The race for sponsored links: bidding patterns for search advertising. Market. Sci. 29 (2), 199–215.
Liu, D., Chen, J., Whinston, A.B., 2010. Ex-ante information and design of keyword auctions. Inf. Syst. Res. 21 (1), 133–153.
Liu, D., Viswanathan, S., 2014. Information asymmetry and hybrid advertising. J. Market. Res. 51 (5), 609–624.
Lu, S., Zhu, Y., Dukes, A., 2015. Position auctions with budget constraints: implications for advertisers and publishers. Market. Sci. 34 (6), 897–905.
Moraga-Gonzalez, J.L., Sandor, Z., Wildenbeest, M.R., 2021. Simultaneous search for differentiated products: the impact of search costs and firm prominence. Econ. J. 131 (635), 1308–1330.
Motta, M., 2004. Competition policy: Theory and practice. Cambridge University Press.
Narasimhan, C., 1988. Competitive promotional strategies. J. Bus. 61 (4), 427–449.
OECD, 2018. Personalised pricing in the digital era. https://one.oecd.org/document/DAF/COMP(2018)13/en/pdf.
Rhodes, A., 2011. Can prominence matter even in an almost frictionless market. Econ. J. 121 (556), 297–308.
Sayedi, A., Jerath, K., Srinivasan, K., 2014. Competitive poaching in sponsored search advertising and its strategic impact on traditional advertising. Market. Sci. 33 (4), 586–608.
Shaked, A., Sutton, J., 1982. Relaxing price competition through product differentiation. Rev. Econ. Stud. 49 (1), 3–13.
Shin, J., Sudhir, K., 2010. A customer management dilemma: when is it profitable to reward one's own customers? Market. Sci. 29 (4), 671–689.
Shin, W., 2015. Keyword search advertising and limited budgets. Market. Sci. 34 (6), 882–896.
Varian, H.R., 1980. A model of sales. Am. Econ. Rev. 70 (4), 651–659.
Varian, H.R., 2007. Position auction. Int. J. Ind. Org. 25 (6), 1163–1178.
Villas-Boas, J.M., 1999. Dynamic competition with customer recognition. Rand J. Econ. 30 (4), 604–631.
Xu, L., Chen, J., Whinston, A.B., 2011. Price competition and endogenous value in search advertising. J. Market. Res. 48 (3), 566–586.
Yang, S., Lu, S., Lu, X., 2014. Modeling competition and its impact on paid-search advertising. Market. Sci. 33 (1), 134–153.
Yao, S., Mela, C.F., 2011. A dynamic model of sponsored search advertising. Market. Sci. 30 (3), 447–468.
Yu, H., Liu, L., Chen, Y., 2018. Strategic product spotlighting on online shopping marketplace: product market and search advertising market. working paper.
Zhang, J., He, X., 2019. Targeted advertising by asymmetric firms. Omega (Westport) 89, 136–150.
Zhou, J., 2011. Ordered search in differentiated markets. Int. J. Ind. Org. 29, 253–262.
Zhu, Y., Wilbur, K.C., 2011. Hybrid advertising auctions. Market. Sci. 30 (2), 249–273.